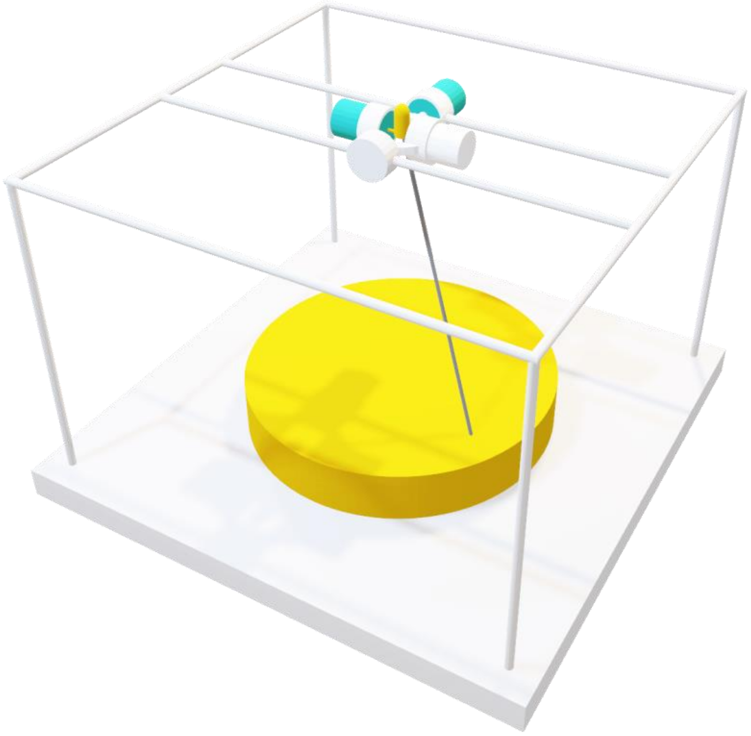
ELEC 341 Design Project

Selective Laser Sintering 3D Printer



Part 1

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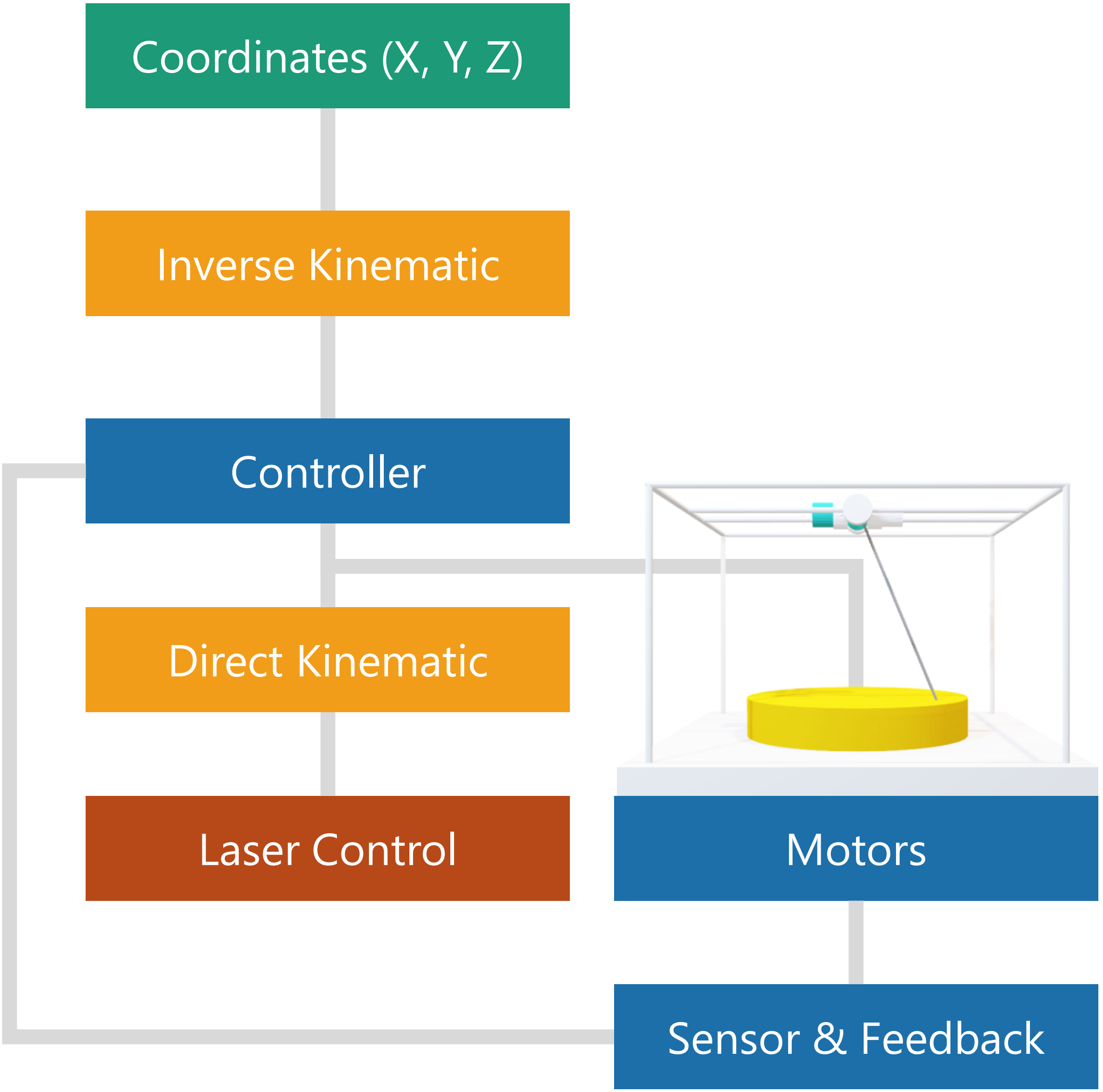


System Overview

The 3D printer uses a laser to build parts. The laser is oriented using a two-axis motor assembly.

The ***inverse kinematic*** module converts the Cartesian coordinates to the desired motor angle.

The ***joint controller*** module handles the system response and controller.

The ***direct kinematic*** converts actual motor angles back to Cartesian coordinates

The ***laser control*** module adjusts the intensity of the laser based on how far the target is

The ***motors*** *control the orientation of the laser*

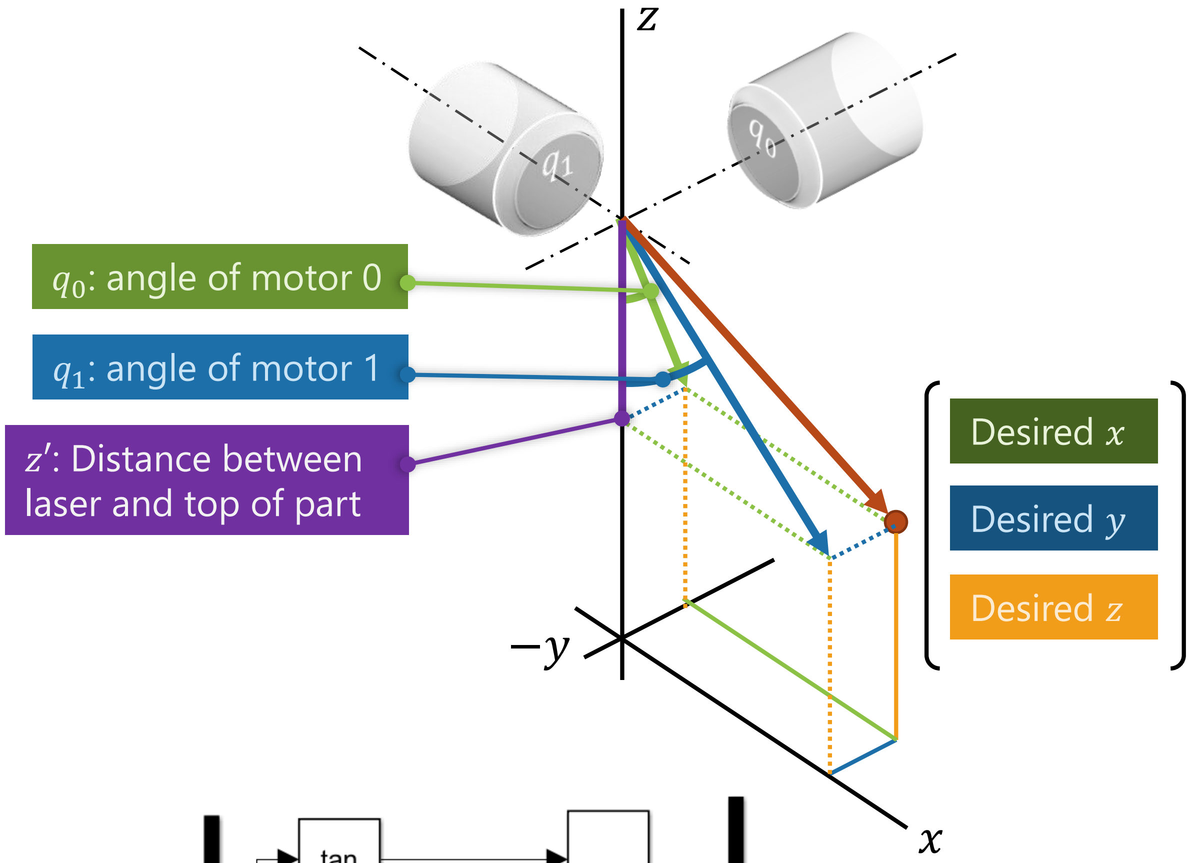
Inverse & Direct Kinematic

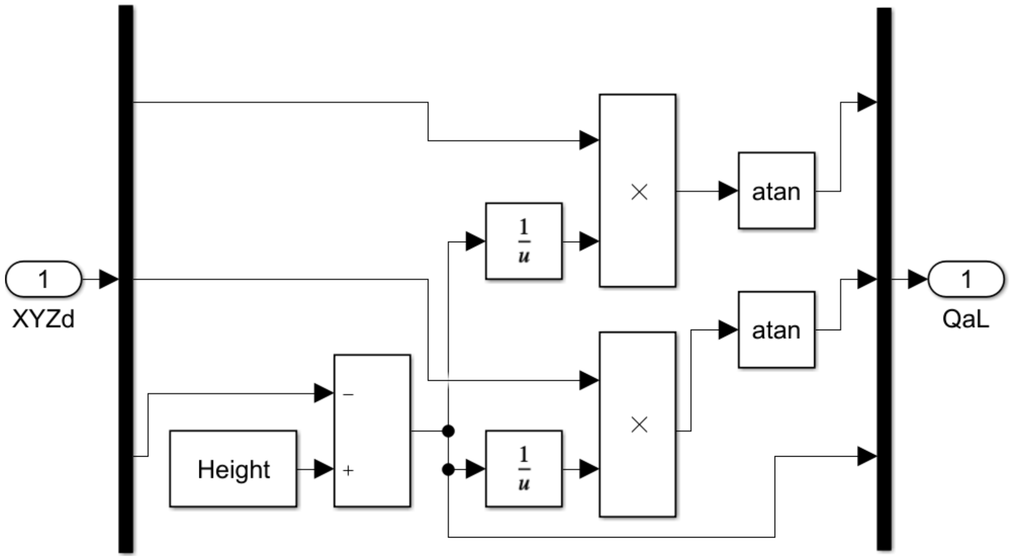
The ***inverse kinematic*** module converts the Cartesian coordinates to the desired motor angle.

$$q_0 = \tan^{-1}\frac{x}{z'} \qquad q_1 = \tan^{-1}\frac{y}{z'} \qquad z' = \text{height} - z$$

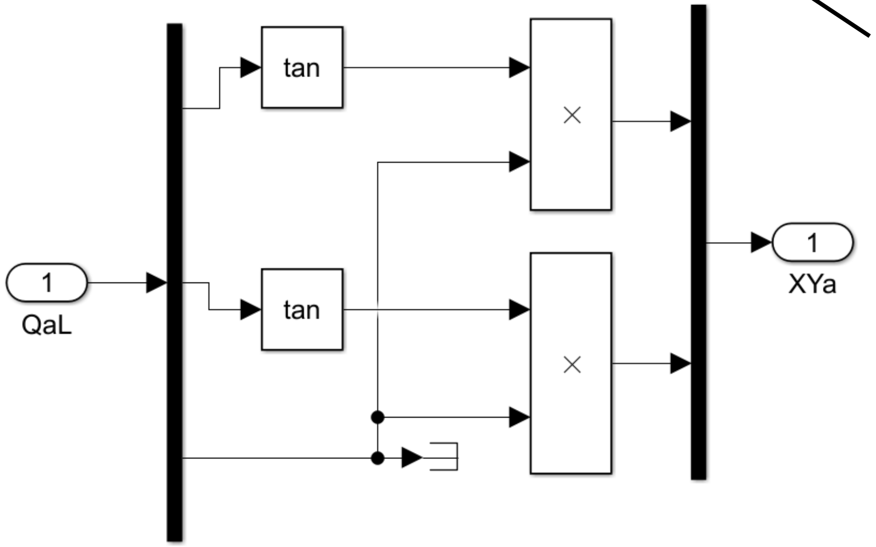
The ***direct kinematic*** converts actual motor angles back to Cartesian coordinates

$$x = z' \tan(q_0)$$
$$y = z' \tan(q_1)$$





I.K. implementation in *SimuLink*



D.K. implementation in *SimuLink*

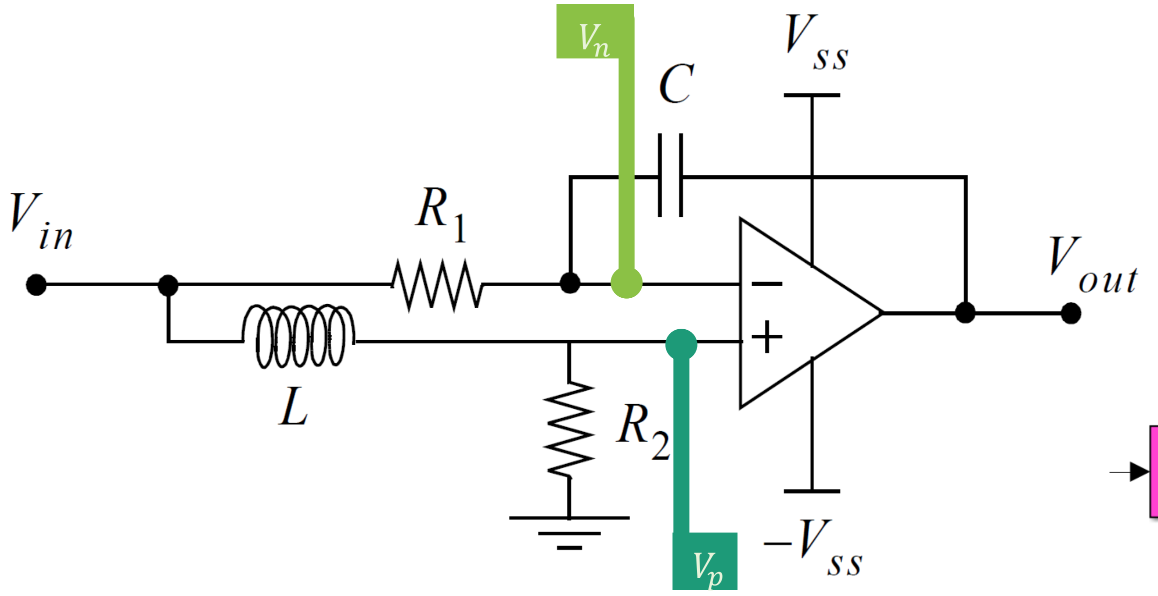
Power Amplifier

The ***power amplifier*** takes the role of a filter and a voltage follower.

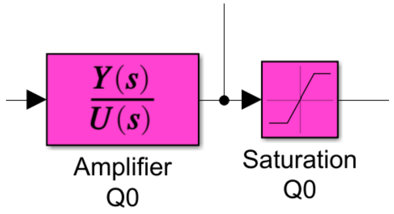
The transfer function is the output divided by input.

Since there is negative feedback coupled by the capacitor, $V_n = V_p$.

MNA is used to solve the circuit



Amplifier circuit



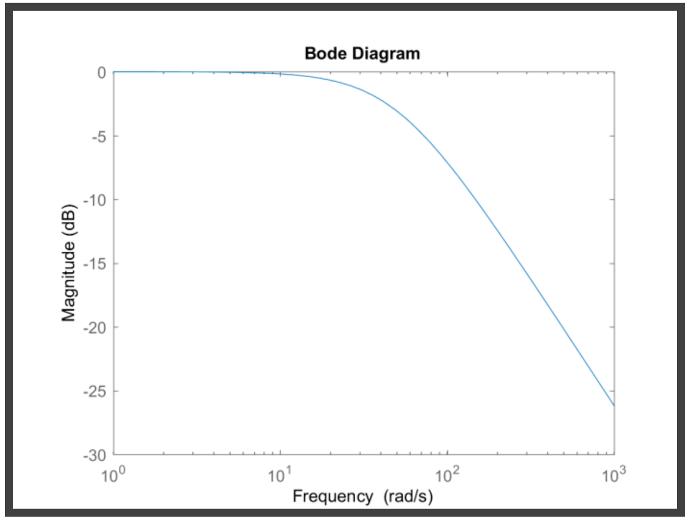
SimuLink implementation

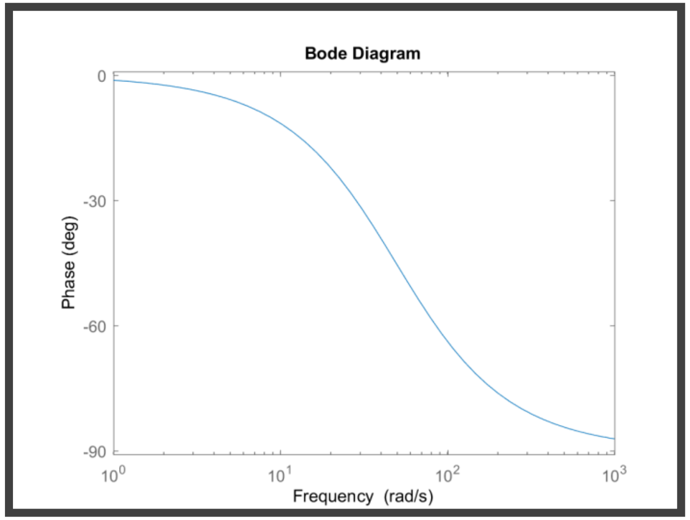
MNA:

$$\frac{V_{in} - V_n}{Ls} = \frac{V_n}{R_2}$$

$$\frac{V_{in} - V_n}{R_1} = (V_n - V_{out}) \times sC$$

Transfer Function: $\frac{Y(s)}{U(s)} = \frac{V_{out}}{V_{in}} = \frac{CR_2R_1 - L}{LCR_1s + CR_1R_2}$





Electric Motor Dynamics

The DC motor has relationships between the torque and the current

$$\tau = K_\tau i$$

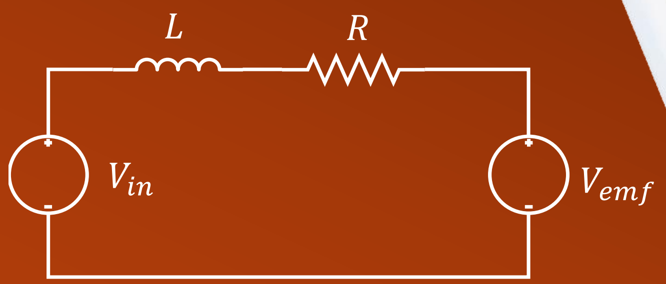
The current is given by Ohm's law

$$i = \frac{V_{in} - V_{emf}}{Ls + R}$$

The back-EMF is proportional to the motor speed

$$V_{emf} = K_b \omega$$

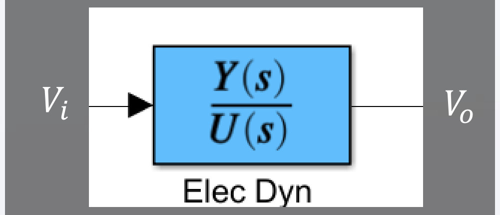
The two motors for q_0 and q_1 are identical, so the relationships are the same for both motors



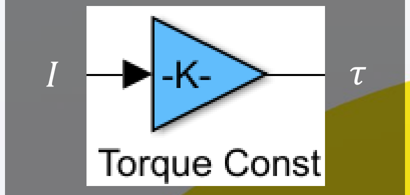
Equivalent circuit

$$\frac{Y(s)}{U(s)} = \frac{1}{Ls + R}$$

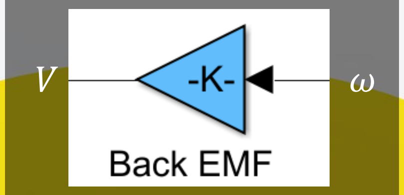
Implementation

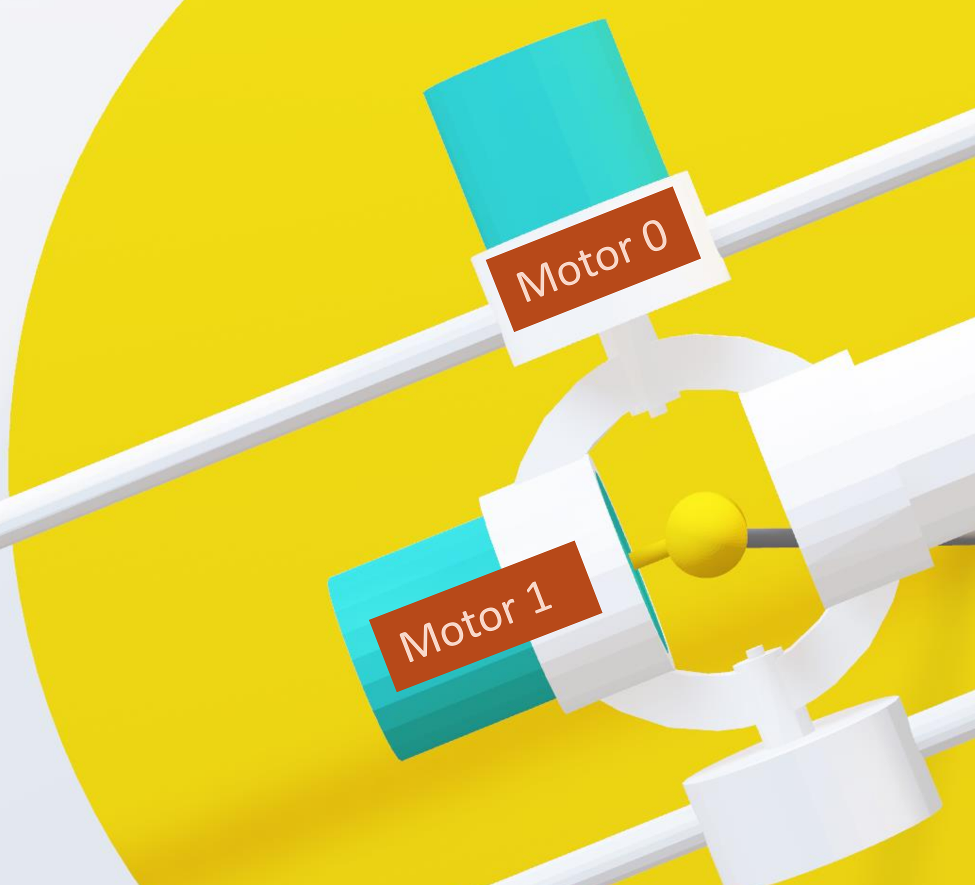


Torque Gain

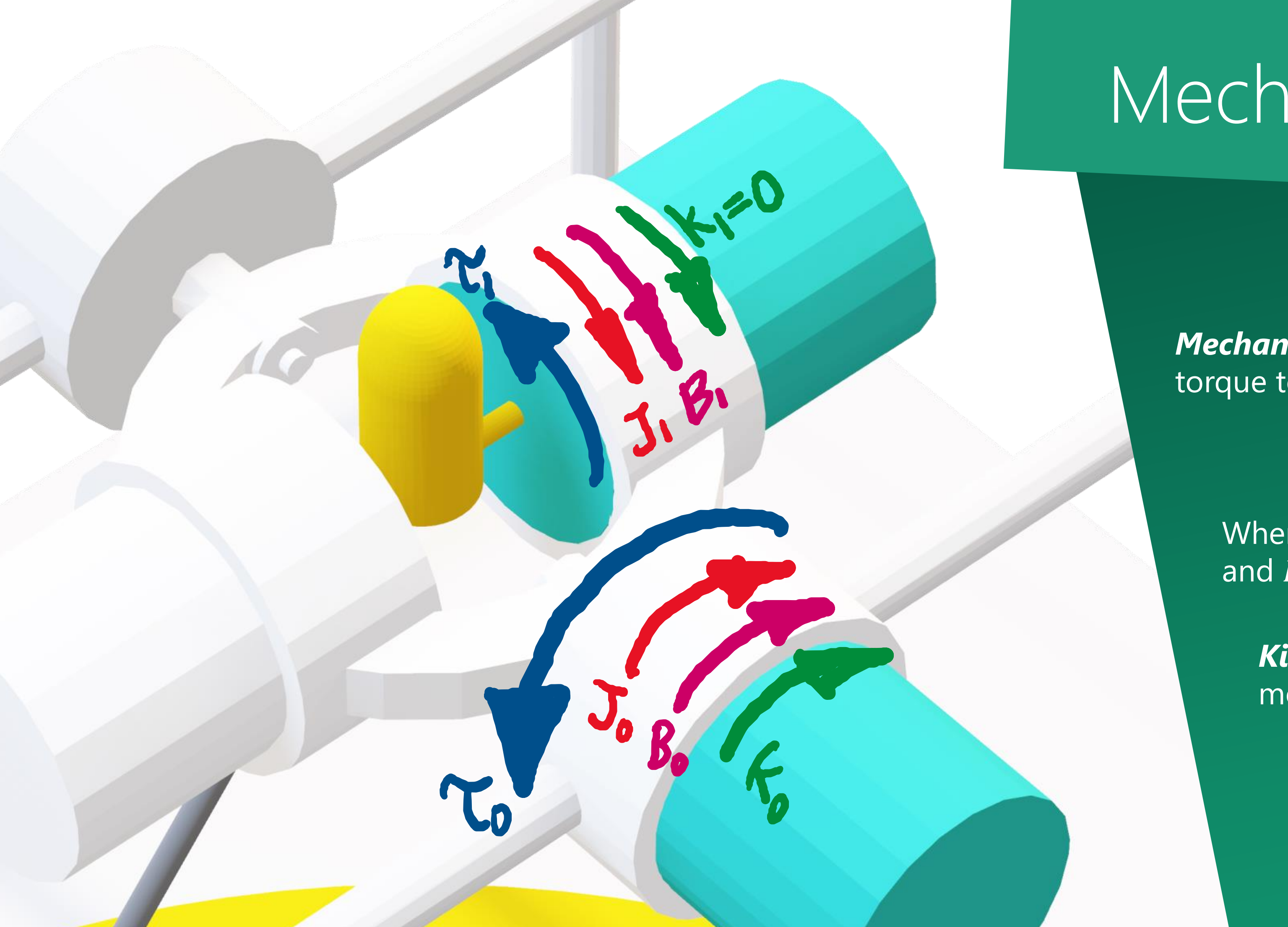


Back EMF Gain

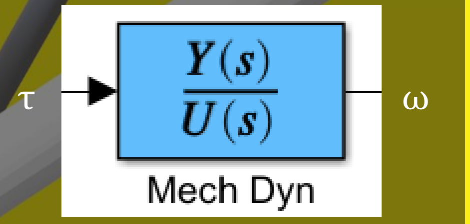




Mechanical Motor Dynamics



Implementation



Mechanical motor dynamics is a transfer function that converts torque to angular speed. It is given as:

$$\frac{Y(s)}{U(s)} = \frac{\omega}{\tau} = \frac{s}{Js^2 + Bs + K}$$

Where J is moment of inertia, B is the kinetic friction constant, and K is the spring constant.

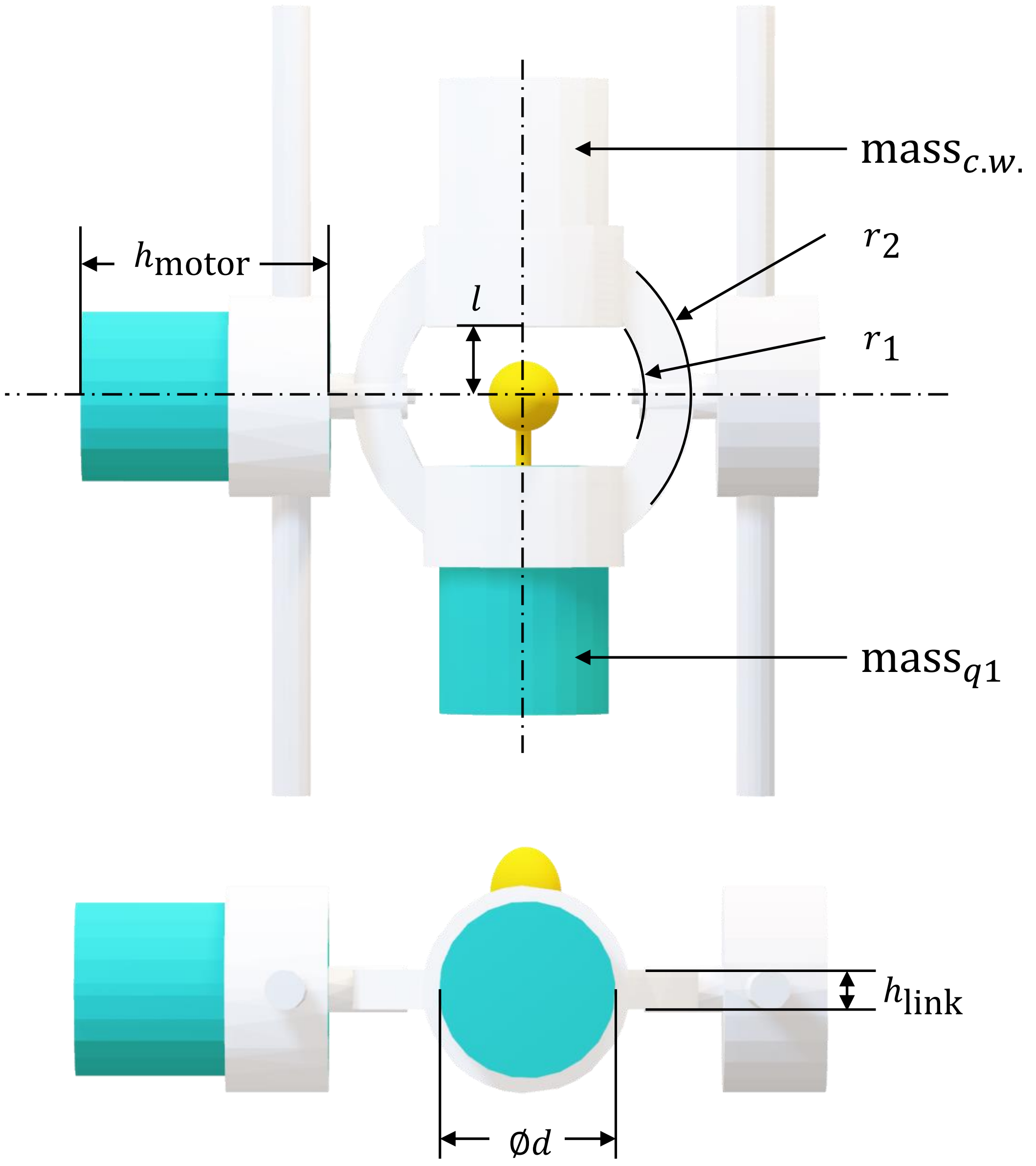
Kinetic friction is the due to the mass imposed on the motion given as

$$B = \frac{I_{\text{noLoad}} K_\tau}{\omega_{\text{noLoad}}}$$

Spring constant is 0 for motor 1 and positive for motor 0

Moment of inertia for motor 1 is J_{rotor} since the laser has negligible mass

$$J_{q1} = J_{\text{rotor}}$$



Mechanical Motor Dynamics

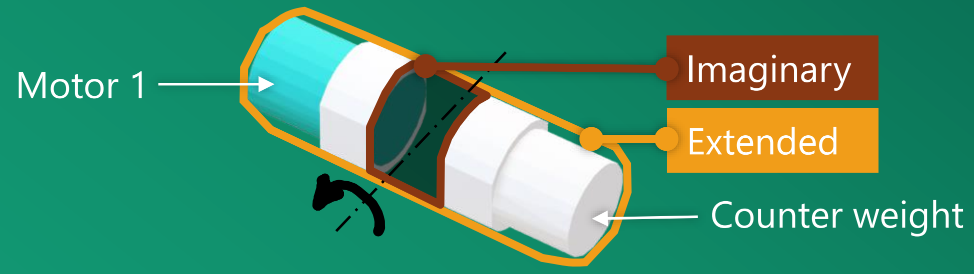
The ***moment of inertia*** component for motor 1 is the superposition of four parts

Motor 0 rotor: Inertia due to the internal rotor of the outer motor

Aluminium link: Hollow cylinder. Mass is its volume multiplied by 6061 aluminium density

$$J_{\text{link}} = \frac{\text{mass}_{\text{link}}}{12}\left(3({r_2}^2 + {r_1}^2) + {h_{\text{link}}}^2\right)$$

Motor 1 & counter weight: $\text{mass}_{c.w.} = \text{mass}_{q1}$



$$J_{q1\,\text{weight}} + J_{\text{counter weight}} = J_{\text{extended}} - J_{\text{imaginary}}$$

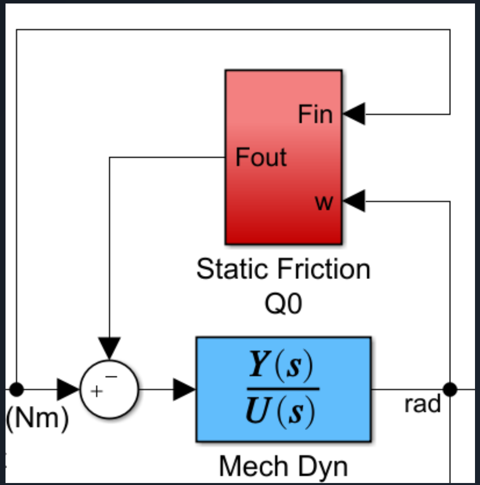
$$J_{q0} = J_{\text{link}} + J_{\text{rotor}} + \left(J_{q0\,\text{weight}} + J_{\text{counter weight}}\right)$$

Static Friction

Static friction works against the applied force and turns into dynamic friction after motor starts moving. It is given by:

$$F_{static} = \mu_{sF} F_N$$
$$F_{static} = \mu_{sF}\, g\,(\mathrm{mass}_{\mathrm{link}} + \mathrm{mass}_{\mathrm{q1}} + \mathrm{mass}_{\mathrm{c.w.}})$$

Implementation

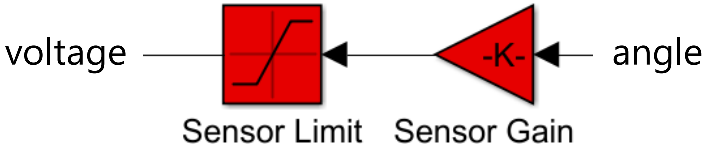


If $\tau_{\mathrm{applied}} < \tau_{\mathrm{static}}$ then $\tau_{\mathrm{net}} = 0$

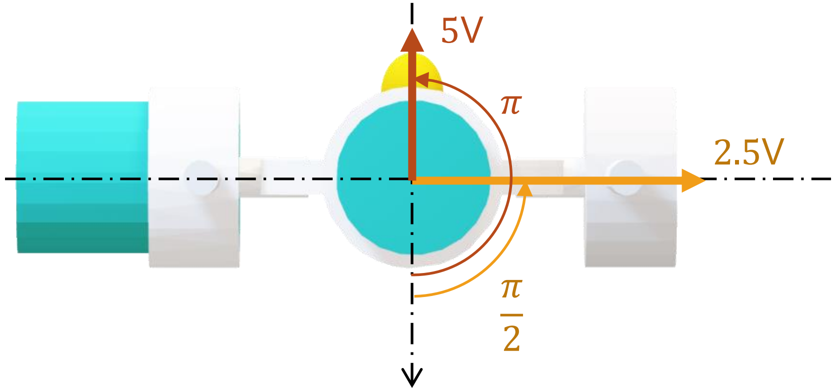
Motor 1 does not experience any noticeable static friction since $\mathrm{mass}_{\mathrm{rotor}} \cong 0$

Sensor Feedback

The ***sensor*** maps the actual angle of the motors to voltage linearly



Gain: $[-\pi, \pi] \rightarrow [-5,5]$

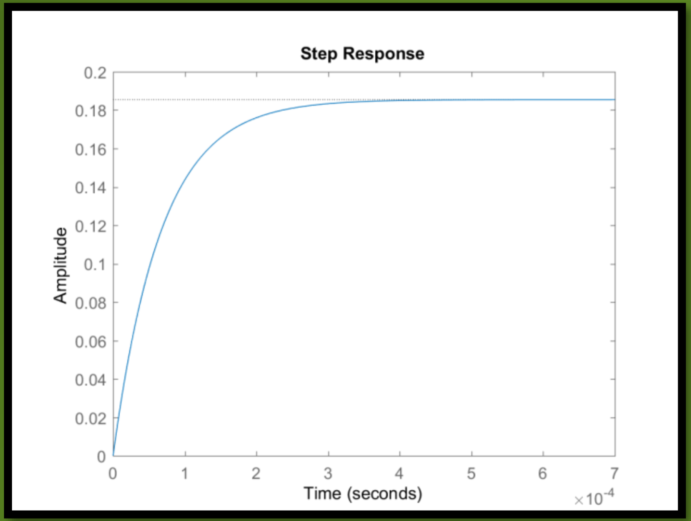


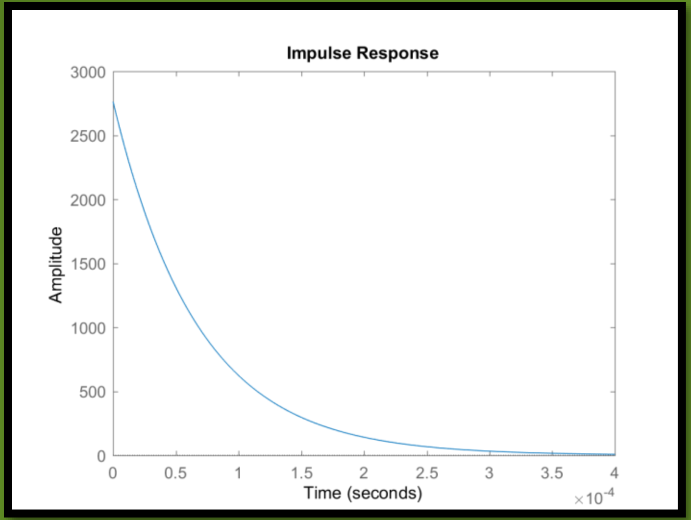
Electrical Response

$$\frac{Y(s)}{U(s)} = \frac{2762.4}{s + 1.489 \times 10^4}$$

Performance

Rise time: 3×10^{-4}s



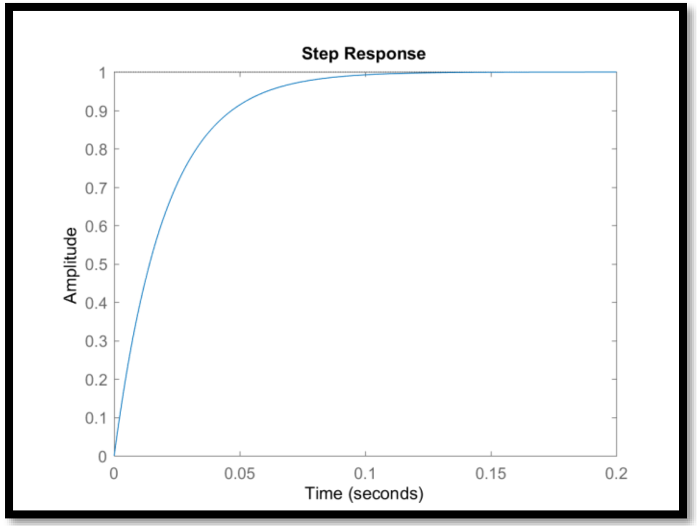


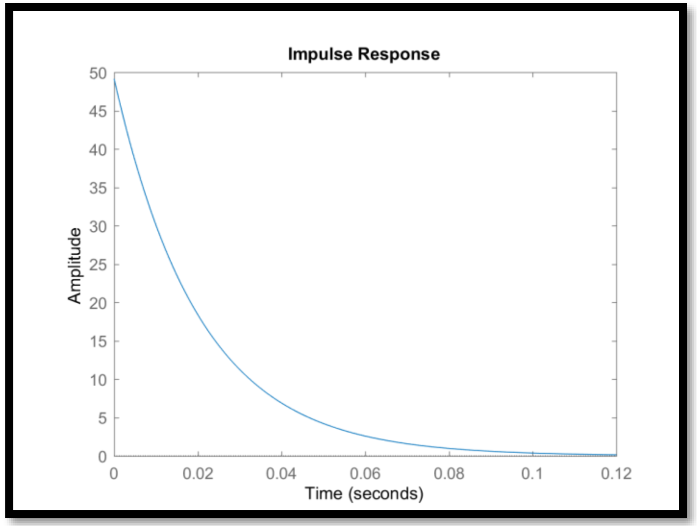
The electrical system produces current given a voltage

Internal inductances of the motors causes exponential behavior

Given a sudden jolt of voltage, the current spikes, but quickly dissipates

Amplifier Response





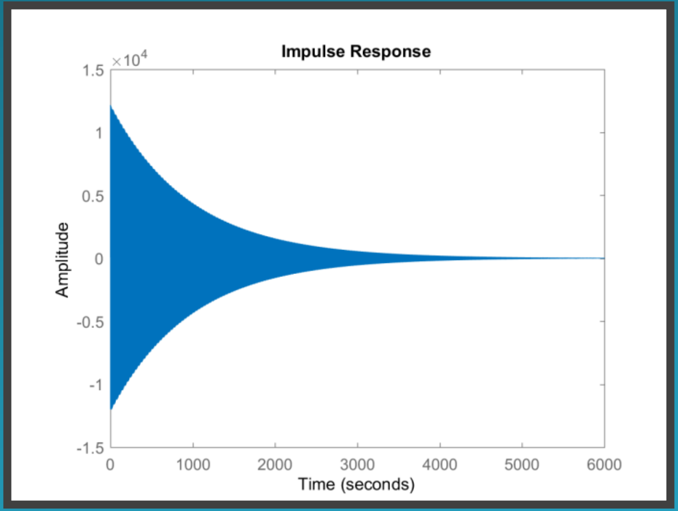
$$\frac{Y(s)}{U(s)} = \frac{49.14}{s + 41.17}$$

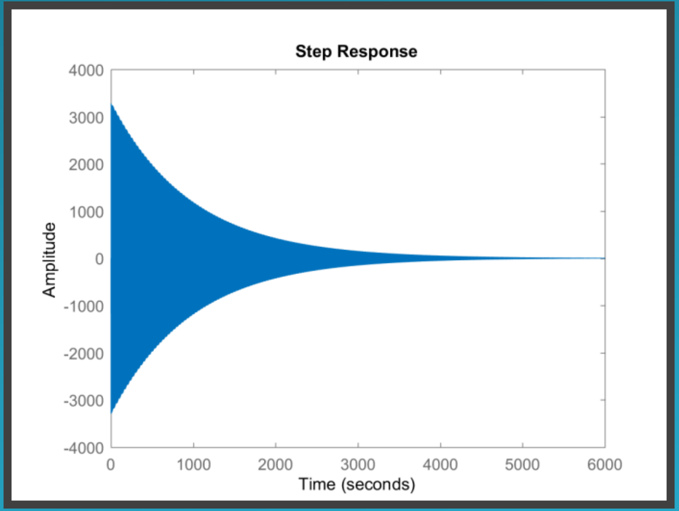
Performance

Rise time: 0.1s

The step response suggests that the integrating amplifier act as a voltage follower with a gain of 1 $\frac{V}{V}$

Motor 0 Response





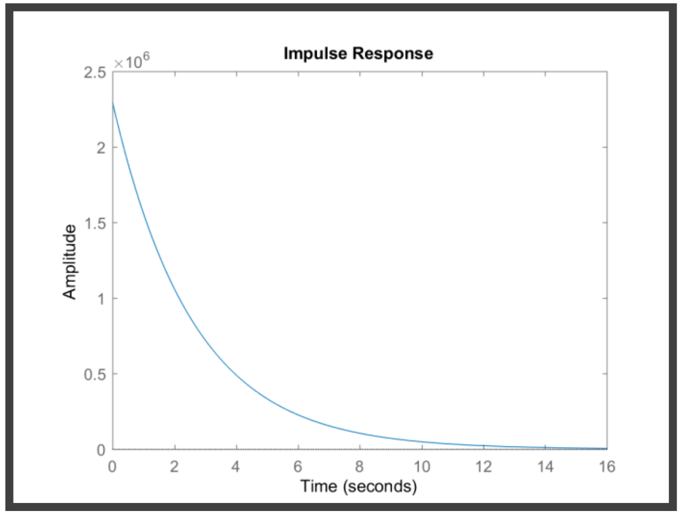
$$\frac{Y(s)}{U(s)} = \frac{12644s}{s^2 + 0.00213s + 14.11}$$

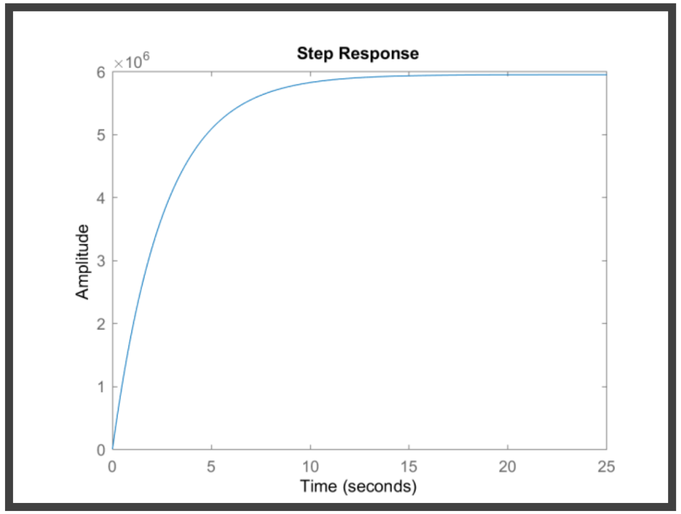
Motor 0 transfer function outputs angular velocity given some torque

Oscillation occurs due to the spring behavior

Angular velocity decays to 0 due to friction

Motor 1 Response





$$\frac{Y(s)}{U(s)} = \frac{2.294 \times 10^6 s}{s(s + 0.3856)}$$

Given a impulse torque, the motor 1 angular velocity decays exponentially due to friction

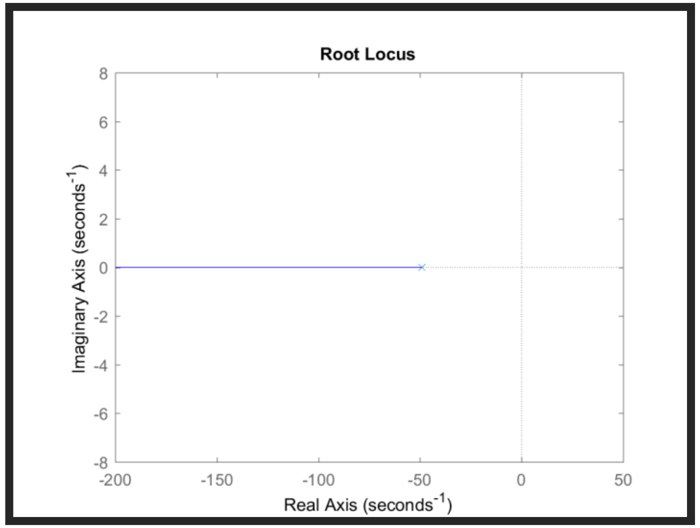
With a constant torque, motor 1 speeds up to its maximum angular velocity

Root Locus

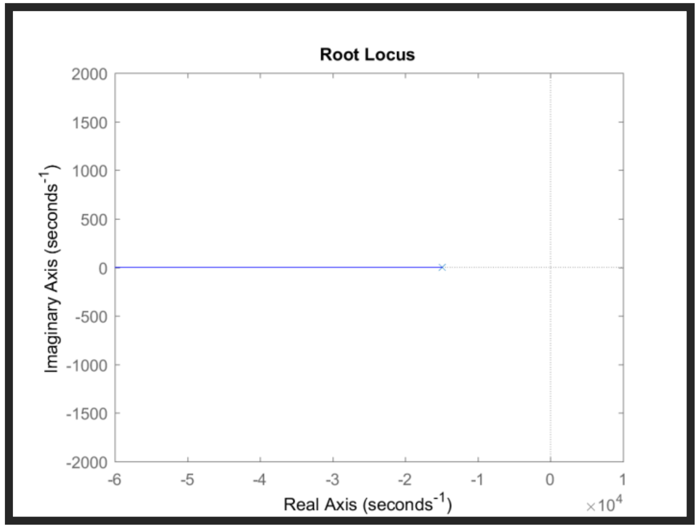
The electrical system, amplifier , and motor 1 are stable according to the root locus

Motor 0 has complex root locus, which explains the damped oscillation

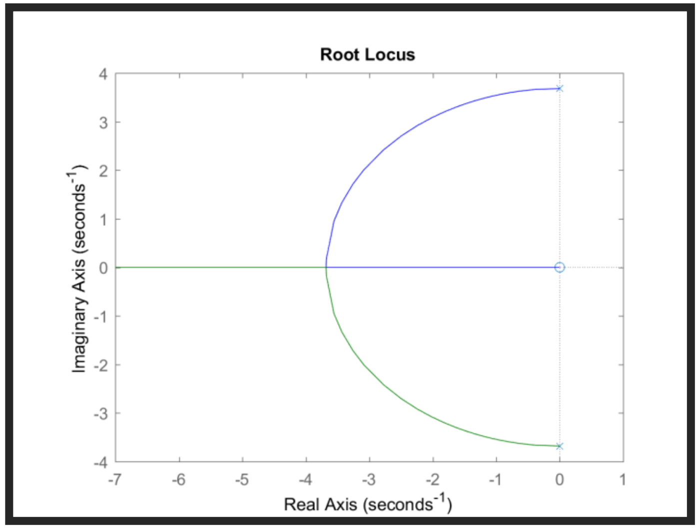
To be continued in part 2 (PID Control)



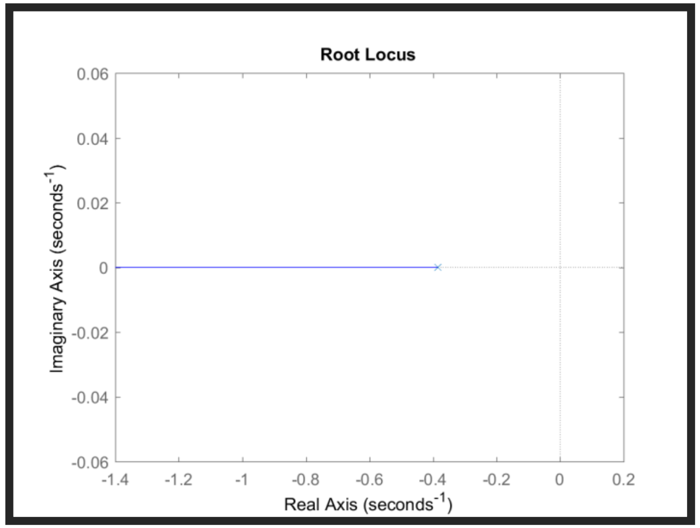
Power amplifier



Electrical



Motor 0



Motor 1