Mini Project 4
Active Filters, Oscillators, and Feedback Amplifiers

ELEC 301
University of British Columbia

Muchen He
44638154
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0 Part A - Active Filter

In this part, we build an active filter using the UA741 op-amps in CircuitMaker (figure 1). In particular, this will be a second order low-pass Butterworth filter. Where $R_1 + R_2 = 10k\Omega$.
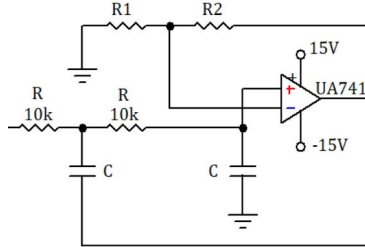
Figure 1: 2nd order Butterworth low-pass filter

The transfer function of this circuit is given as:

$$H(s) = A_M \frac{\frac{1}{(RC)^2}}{s^2 + s\frac{3-A_M}{RC} + \frac{1}{(RC)^2}}$$

Where A_M is the pass-band gain, and the fractional part is in unity gain, and $A_M = 1 + \frac{R_2}{R_1}$.

Since we are interested in the poles of the transfer function as it contributes to the pole locations and ultimately define the system behaviour, we rewrite the denominator as

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

Where ζ is the damping constant, and ω_n is the natural frequency of the system with no damping. It follows that

$$\omega_n = \frac{1}{RC}, \quad \zeta = \frac{3-A_M}{2}$$

0.1 Capacitance and Gain

First, we want the 3dB frequency to be at 10kHz, or $\omega_c = 2\pi \times 10\text{krad s}^{-1}$. The cut-off frequency is given as $\omega_c = \frac{1}{RC}$. Knowing that $R = 10k\Omega$, we solve for C, which we obtain

$$C = 1.6\text{nF}$$

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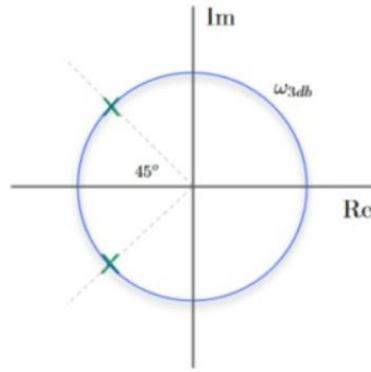
Figure 2: 2nd order Butterworth poles on the s-plane

We want to find the values A_M which will make the above circuit into a Butterworth filter. A second order Butterworth filter, by definition the poles 45 deg apart from the negative imaginary axis on the s-plane (see figure 2). The corresponding normalized Butterworth polynomials are $s^2 + \sqrt{2}s + 1$. In other words $\sqrt{2} = 3 - A_M$. It follows that

$$A_M = 3 - \sqrt{2} = 1.586$$

The corresponding resistor values are therefore as follows.

$$R_1 = 3.868\text{k}\Omega, \quad R_2 = 6.132\text{k}\Omega$$

Using these values, the bode plot of the filter is as follows, in figure 3.

Figure 3: Magnitude and phase bode plots of the second order Butterworth filter

On the s-plane, the two poles sit on the the circle with a radius of the frequency $\omega_c = \frac{1}{RC} = 62500\text{rad}\,\text{s}^{-1}$. As seen on the root locus graph in figure 4a, the poles are where the root locus contour starts (at 45 deg from the negative real axis).

(a) Pole locations   (b) Oscillating pole locations

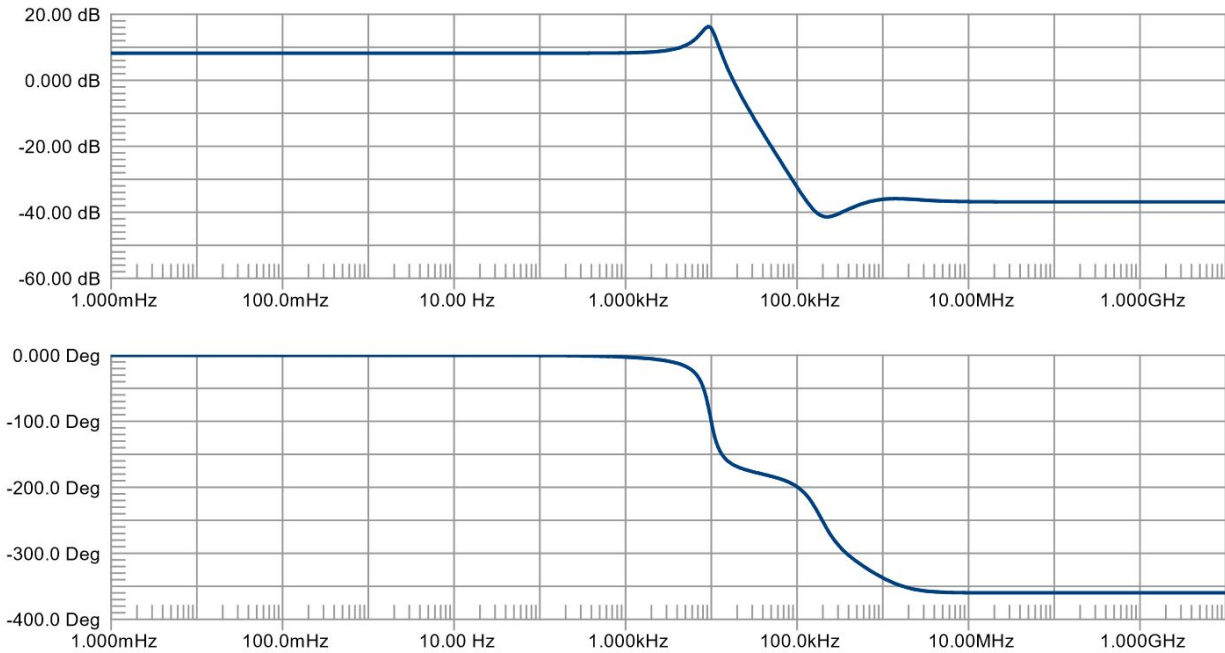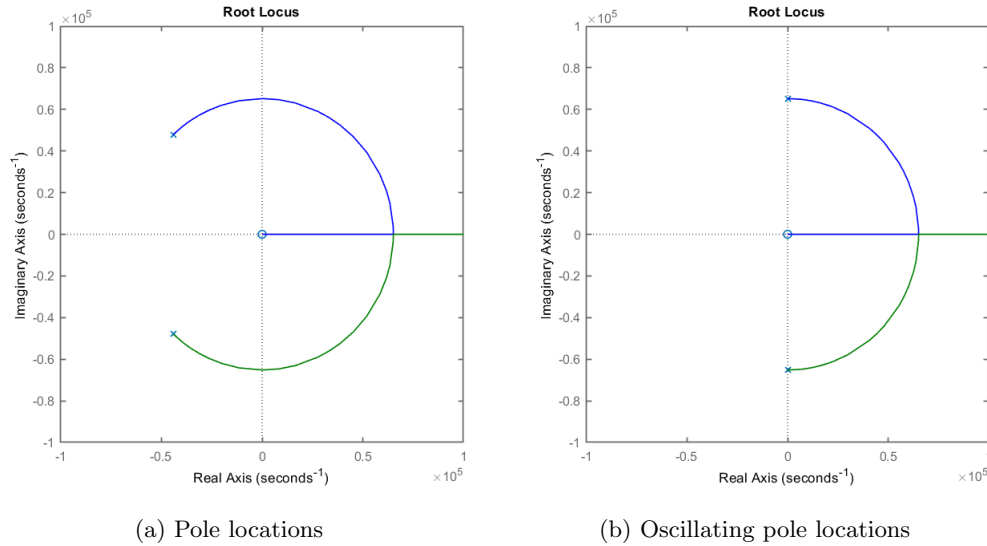## 0.2   Oscillation Gain

While keeping R_1 and R_2 sum the same at 10kΩ, A_M is slowly increased. At A_M=3, the 's' term in the denominator of the transfer function becomes 0. This is when the two poles sit on the imaginary axis (as seen in figure 4b. The system is oscillating with a frequency of 9.5kHz and marginally stable. The output of the filter is as shown in figure 5.
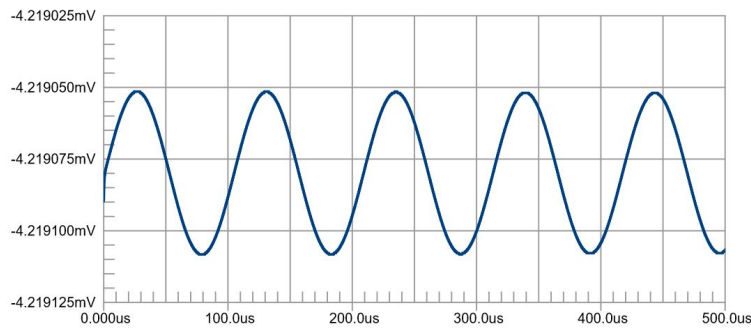
Figure 5: Oscillating output of Butterworth filter

As A_M increases, the poles move along the root locus (the circle) from the pole location as shown in figure 4a to the right until it lands on the imaginary axis as shown i figure 4b. At which point, the response is a undamped oscillation - even with input shorted (input signal is zero). Increasing A_M further will cause the filter to go unstable as the poles have now crossed to the right-hand plane.

1   Part B - Phase Shift Oscillator

The circuit as shown in figure 6 is built using UA741 OP-AMP, and $R = 1$kΩ, $C = 1$μF.
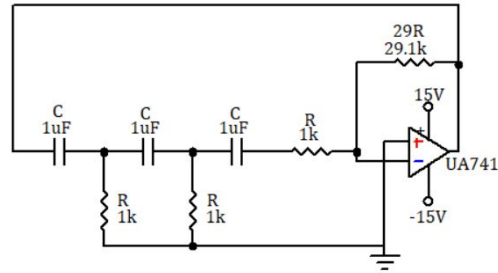
Figure 6: Phase shift oscillator circuit

The oscillators have the denominator of the transfer function is a pair of complex poles that sit on the imaginary axis, with $\sigma = 0$. Thus, a finite signal is produced even when there is no input signal.

The feedback resistor initial had a resistance of $29R = 29\text{k}\Omega$. Even though the circuit produces an oscillation, it is found that the signal would eventually decay. Thus, the resistance is increased to $29.1\text{k}\Omega$ so the signal would never "die" out.

The values of C and R is halved, then doubled. Table 1 shows different frequencies the circuit generated.

| Circuit	Original	Halved	Doubled
R	1kΩ	0.5kΩ	2kΩ
C	1µF	0.5µF	2µF
Frequency)	64.7Hz	258Hz	16.2Hz

Table 1: Frequencies at different R and C values

Calculated Frequencies
We can calculate the expected frequency manually using the following equation.

$$\omega = \frac{1}{\sqrt{6}RC}, \quad f = \frac{1}{2\pi\sqrt{6}RC}$$

Plugging in the original resistor and capacitor values, which was $R = 1\text{k}\Omega$ and $C = 1\text{µF}$, we obtain $f = 64.974\text{Hz}$.

Notice that the equation only depends on R and C; if R and C are both doubled, denominator is quadrupled, thus the frequency is reduced by a factor of four. Thus when we double the capacitor and resistor values, the frequency is $f = 16.244\text{Hz}$.

Similarly, when we half both R and C, we should expect the frequency to be 4 times the original. Indeed it is: $f = 259.9\text{Hz}$.

There are some discrepancies between the calculated and measured value. However, the error is insignificant. This could be due to human error while measuring the frequency. It could also attribute to the non-idealities of real OP-AMP components.

2 Part C - Feedback Amplifier

The feedback circuit is built using common collector amplifier with BJTs. The BJT used is the 2N3904 type. A feedback resistor is placed from the output node to the input node. The circuit is as shown in figure 7
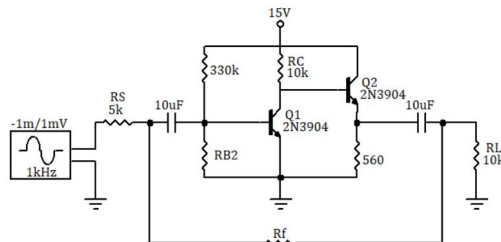
Figure 7: Feedback amplifier circuit

In order to find R_{B2} , we do a parameter sweep on the resistance value. As seen from the transient response in figure 8, an R_{B2} with value around 20kΩ yields maximum gain.
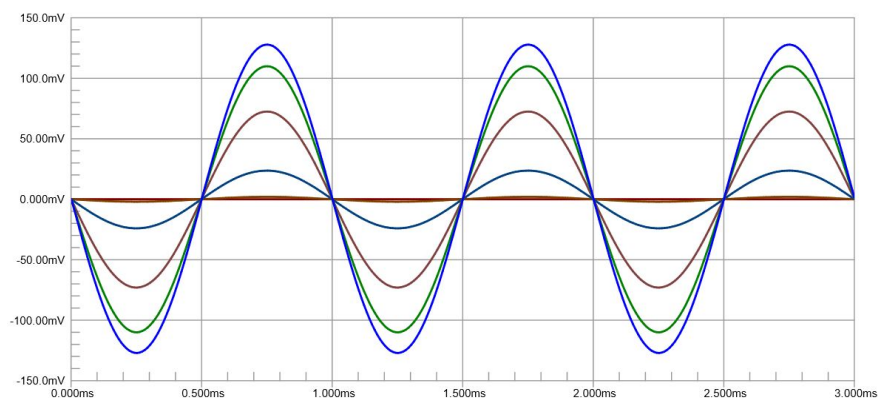
Figure 8: Parameter sweep of R_{B2}

Some additional information to note is that the feedback network in this feedback amplifier has a **shunt-shunt** topology. This is because the output voltage is sampled, and is controlling the input current.

2.1 DC Operating Point

With the infinity capacitors decoupled, we have the DC bias circuit for the transistors. The voltage and currents of both transistors are as follows in table 2.

	$\mathbf{V_C}$	$\mathbf{V_B}$	$\mathbf{V_E}$	$\mathbf{I_C}$	$\mathbf{I_B}$	$\mathbf{I_E}$
Q1	1.90V	0.654V	0.00V	1.295mA	10.77µA	1.305mA
Q2	15.0V	1.90V	1.26V	2.19mA	15.4µA	2.21mA

Table 2: Q1 and Q2 DC operating points

For transistor parameters, we utilize the following equations:

$$h_{fe} \approx h_{FE} = \frac{I_C}{I_B}, \quad g_m = \frac{I_C}{V_T}, \quad r_\pi = \frac{h_{fe}}{g_m}$$

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Thus, the parameters for Q1 and Q2 is as follows:

	$\mathbf{h_{fe}}$	$\mathbf{g_m}$	r_π
Q1	120	0.052	2.32kΩ
Q2	142	0.088	1.62kΩ

Table 3: Q1 and Q2 transistor parameters

2.2 Open Loop Response

Setting the feedback resistor to an open circuit (infinite resistance), we obtain the open loop response from the circuit. The amplitude bode plot depicts the amplitude open loop response, shown in figure 9.

From the plot, we identify the low and high 3dB frequencies: $\omega_{L3dB} = 2.883$Hz and $\omega_{H3dB} = 89.63$kHz. Furthermore, we get a midband gain of 42.1dB. or 127.4 V/V. But since this is an inverting amplifier, the voltage gain is actually -127.4 V/V.
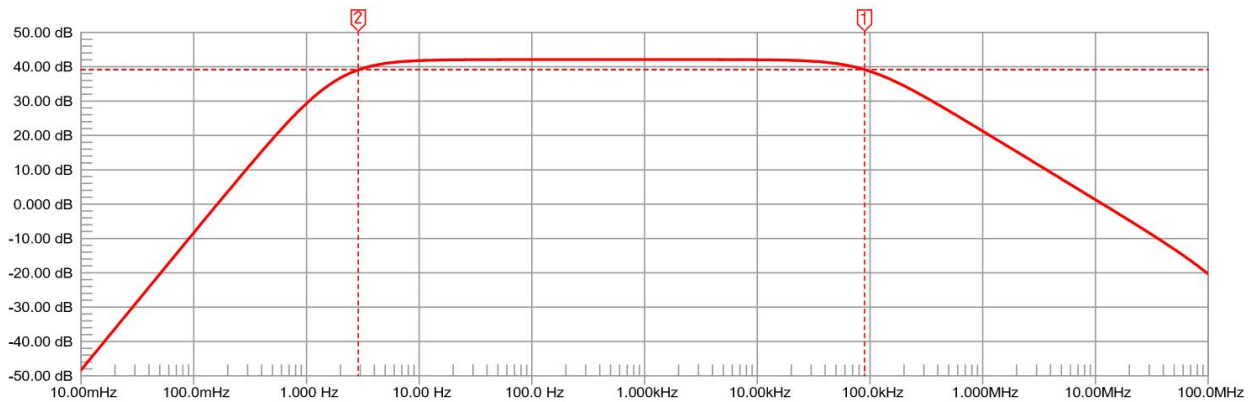
Figure 9: Open loop amplitude bode plot of the feedback amplifier

Applying a test voltage source and measuring the test current, we can find the input and output impedance.

The measured input impedance is $\frac{V_{\text{test}}}{I_{\text{test}}} = \frac{240.4\mu V}{93.35nA} = 2.575$kΩ.
The measured output impedance is $\frac{V_{\text{test}}}{I_{\text{test}}} = \frac{707.1\mu V}{11.18\mu A} = 63.247$Ω.

2.2.1 Predicting Closed Loop Response

In order to obtain closed loop prediction, we must analyse the feedback network and the small signal model in greater detail. First, we know that the feedback network (figure 10) utilizes shunt-shunt topology; for that reason, we will use **y-parameters** to represent the equivalent circuit of the feedback network.

Figure 10: Feedback network of the feedback amplifier

Where y-parameters uses these set of equations:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Now we compute for the y-parameter matrix.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{R_f}, \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{1}{R_f}$$
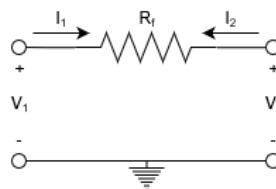
Because the effect due to the feed-forward gain of y_{21} is so small, the gain may be neglected. Thus we proceed to compute for

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{R_f}$$

Voltage Gain

For R_f=100kΩ, we obtain the feedback gain $\beta = y_{12} = -\frac{1}{R_f} = -10\mu S$. Recall the open loop voltage gain is -127.4 V/V. However, since we are using a shunt-shunt topology, we need a current controlled voltage amplifier, thus we need the gain in terms of V/I. In particular we want the gain in terms of V_{out}/i_i. Hence,

$$\frac{V_{out}}{i_i} = \frac{V_{out}}{\frac{V_i}{R_s}} = R_s\left(\frac{V_{out}}{V_i}\right)$$

Where $R_s = 5k\Omega$. Plugging everything in, we obtain the open loop gain $A = 5k\Omega \times -127.4V/V = -636.75kV/A$. This makes sense because the feedback gain is in the units of Siemens, thus we need the open loop gain in the units of ohms, which is V/A.

Using the feedback equation we evaluate for the closed loop gain.

$$\begin{aligned} A_f &= \frac{A}{1 + A\beta} \\ &= \frac{636752}{1 + -636752 \cdot -1 \times 10^{-5}} \\ &= 86.43kV/A \end{aligned}$$

Converting back to voltage, we use the relationship in the above equation. Finally, the voltage gain with feedback is $-17.285V/V$.

3dB Frequencies

Recall the open-loop frequencies are $\omega_{L3dB} = 2.883Hz$ and $\omega_{H3dB} = 89.63kHz$. Due to the feedback, there exists a bandwidth extension with a factor of $(1 + A\beta)$. In particular,

$$\omega_{L3dB_f} = \frac{\omega_{L3dB}}{1 + A\beta} = \frac{2.883}{1 + (-6.37 \times 10^5)(-1 \times 10^{-5})} = 0.391Hz$$

$$\omega_{H3dB_f} = \omega_{H3dB}(1 + A\beta) = 89.63 \times 10^3(1 + (-6.37 \times 10^5)(-1 \times 10^{-5})) = 660.35kHz$$

I/O Impedances

Recall the open loop input and output impedances are respectively 2.575kΩ and 63.247kΩ. The amplifier utilizes a shunt-shunt topology, which means that the input impedance with feedback is reduced and the output impedance with feedback is also reduced. Specifically, they are reduced by $(1 + A\beta)$. In particular,

$$R_{i_f} = \frac{R_i}{1 + A\beta} = \frac{2.575 \times 10^3}{1 + (-6.37 \times 10^5)(-1 \times 10^{-5})} = 349.5\Omega$$

$$R_{o_f} = \frac{R_o}{1 + A\beta} = \frac{63.247}{1 + (-6.37 \times 10^5)(-1 \times 10^{-5})} = 8.585\Omega$$

2.2.2 Comparing Predicted Values

Simulation in CircuitMaker is ran to verify how close the predicted / calculated values are to the actual results. First, the frequency response:
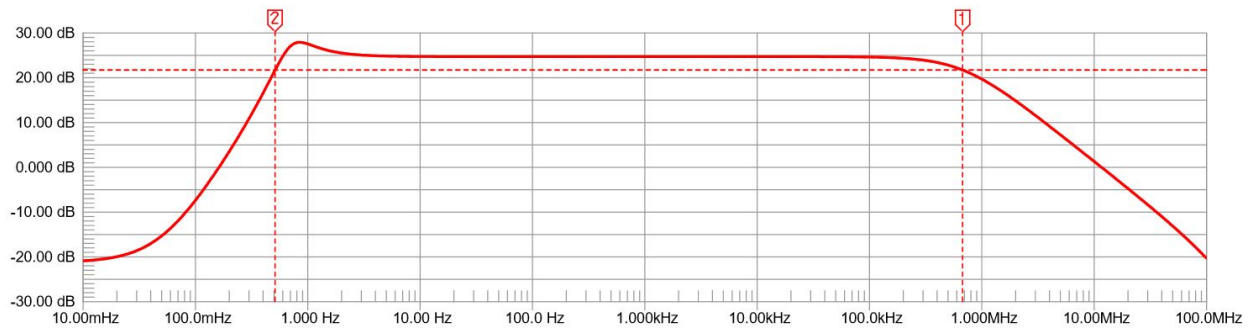

Figure 11: Frequency response of feedback amplifier with R_f of 100kΩ

The mid-band gain at 1kHz is 24.735dB from the bode plot (figure 11), or 17.25 in magnitude. Since it is still an inverting amplifier, the voltage gain is actually -17.25 V/V. Our prediction of -17.285 V/V is excellent.

Next, using the bode plot (figure 11), we obtain the low and high 3dB frequencies. The low 3dB frequency is 0.512Hz and the high 3dB frequency is 676.2kHz. The inaccuracy of the low 3dB frequency could possibly attribute to the peaking of gain at the low-frequency cut off point.

For input and output impedances, test sources are setup in the circuit at the input and output. The test voltage is imposed and the test current is measured. The result is as follows.

$$R_{i_f} = 241\Omega, \quad R_{o_f} = 8.657\Omega$$

The predicted input impedance has a higher error but around the same magnitude. However, the output impedance is accurate.

2.3 Estimating Feedback with Gain

For five different values of R_f: 1kΩ, 10kΩ, 100kΩ, 1MΩ, and 10MΩ, the feedback amplifier is simulated and the frequency response is as follows (figure 12). Where the red plot corresponds to R_f of 1kΩ, blue plot corresponds to 10kΩ, green plot corresponds to 100kΩ, yellow plot corresponds to 1MΩ, and finally the purple plot corresponds to 10MΩ.
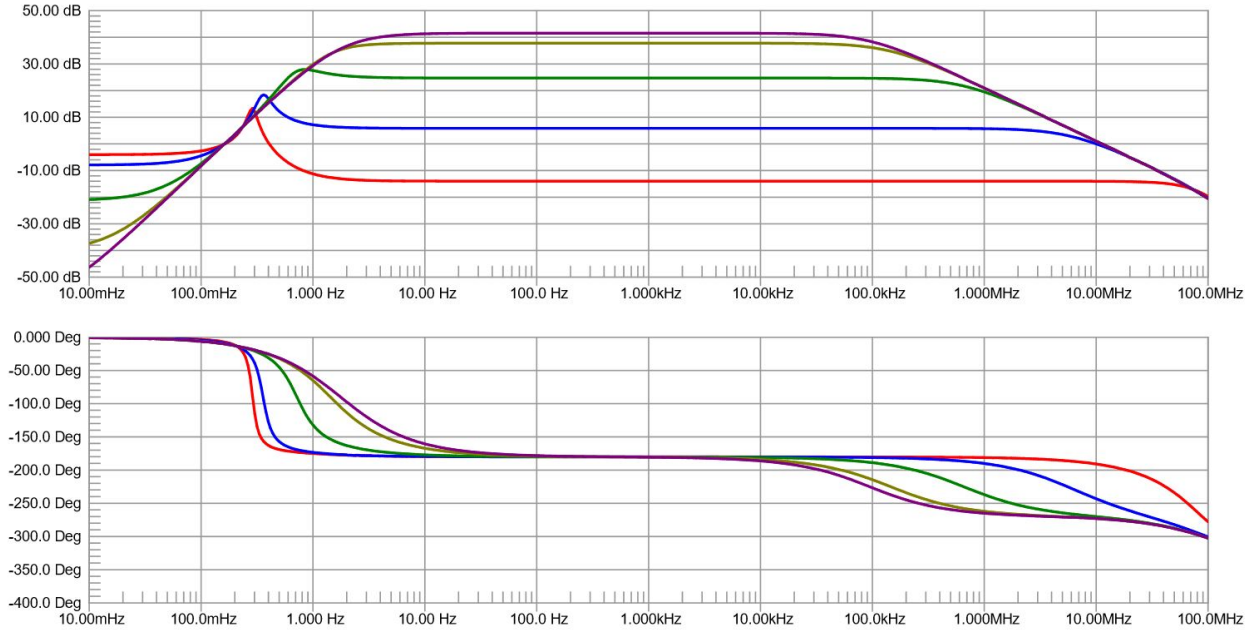
Figure 12: Frequency response of feedback amplifier with various R$_f$

We can compute the feedback gain β just by knowing the mid-band gain with feedback and the open loop gain. Recall the open loop gain is -636.75kV/A.

First, we need to convert midband gain at 1kHz from decibels to V/V. Then V/A because reasons discussed previously. Finally, we solve the equation $A_f = \frac{A}{1+A\beta}$ for β.

Additionally, as discussed earlier, the feedback gain is also calculated as $\beta = -\frac{1}{R_f}$ from the feedback network. Table 4 contains the feedback gain of each configuration and comparison to the calculated β value.

R$_f$ (Ω)	V-gain (dB)	V-gain (V/V)	A$_f$ (V/A)	β (A/V)	$\beta_{\mathbf{calculated}}$ (A/V)
1k	14.02	-0.199	-995.4	-1.00×10^{-3}	-1.00×10^{-3}
10k	5.862	-1.964	-9.82×10^3	-1.00×10^{-4}	-1.00×10^{-4}
100k	24.74	-17.26	-86.29×10^3	-1.00×10^{-5}	-1.00×10^{-5}
1M	37.83	-77.85	-389.2×10^3	-9.99×10^{-7}	-1.00×10^{-6}
10M	41.58	-120.0	-600.0×10^3	9.06×10^{-8}	-1.00×10^{-7}

Table 4: Feedback gains with various feedback resistances

2.4   Estimating Feedback with I/O Impedances

At 1kHz, R$_f$ is varied between 10kΩ, 100kΩ, and 1MΩ. The input and output impedances are measured using techniques described before. They are displayed in table 5.

R$_f$ (Ω)	R$_{i_f}$ (Ω)	R$_{o_f}$ (Ω)
10k	26.46	1.139
100k	241.1	8.650
1M	1.308k	38.43

Table 5: Input and output impedances with varying feedback resistance

With these input and output impedance values, the open loop input and output impedances, and the open loop gain A, we can compute for the amount of feedback / feedback gain, β using the relationships:

$$R_{i_f} = \frac{R_i}{1 + A\beta}, \quad R_{o_f} = \frac{R_o}{1 + A\beta}$$

The feedback gain for each R$_f$ value is in the table as follows (table **??**). The calculated feedback form using the input impedance quite inaccurate compared to the feedback gain calculated from the output impedance. The average of the two is also taken for comparison purposes.

R$_f$ (Ω)	R$_{i_f}$ (Ω)	R$_{o_f}$ (Ω)	β_i (S)	β_o (S)	β_{ave} (S)
10k	26.46	1.139	-1.51×10^{-4}	-8.564×10^{-5}	-1.18×10^{-4}
100k	241.1	8.650	-1.52×10^{-5}	-9.91×10^{-6}	-1.26×10^{-5}
1M	1.308k	38.43	-1.52×10^{-6}	-1.01×10^{-6}	-1.27×10^{-6}

Table 6: Input and output impedances with varying feedback resistance

Compared to the feedback values calculated above in table 4, estimating the feedback gain using input and output impedances are slightly more inaccurate.

2.5 Desensitivity Factor

The gain de-sensitivity can be found by taking the derivative of the feedback gain with respect to the open loop gain.

$$\frac{\mathrm{d}A_f}{\mathrm{d}A} = \frac{1}{(1 + A\beta)^2}$$
$$\frac{\mathrm{d}A_f}{A_f} = \left(\frac{1}{1 + A\beta}\right)\frac{\mathrm{d}A}{A}$$

Thus the desensitivity factor is given by $1 + A\beta$.

When $R_f = \infty$, there is no feedback, and thus the amplifier is open loop. Then obviously the feedback factor $\beta = 0$ and thus the desensitivity factor is 1.

Varying R$_C$ in the amplifier circuit to 9.9kΩ and 10.1kΩ, we see the gain change as follows (table 7).

R$_C$	V-gain (V/V)
9.9k	-126.853
10k	-127.556
10.1k	-128.204

Table 7: Gain of feedback amplifier with varying R$_C$

When R$_f$=100kΩ, however, by varying the R$_C$ as before, we then again observe what happens to the gain when R$_C$ changes (table 8).

R$_C$	V-gain (V/V)
9.9k	-17.235
10k	-17.248
10.1k	-17.260

Table 8: Gain of feedback amplifier with varying R$_C$

We see that the gain barely change, hence desensitized by the feedback network. The desensitization factor is $1 + A\beta = 1 + 7.368$.

References

[1] A. Sedra and K. Smith. "Microelectronic Circuits", 5th, 6th, or 7th Ed. Oxford University Press, New York. Web. 3 November 2017.

[2] N. Jaeger. *ELEC 301 Course Notes*. University of British Columbia. Web. 3 November 2017.

[3] ON Semiconductor: 2N3904 Datasheet. ON Semiconductor. Web. 26 October 2017. `https://www.onsemi.cn/PowerSolutions/document/2N3903-D.PDF`