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# **1. Introduction**

This report depicts the the methods used to solve the problem introduced in this Mini Project. This report also illustrates the validity of the some methods of approximations by using circuit simulation. The objective of this mini project is to practice solving, designing, and simulating circuits, in particular, filters.

# 2. Problems

## Part l

### Α

The transfer function to the given circuit is  $T(s) = 0.25 imes rac{s}{s+5} imes rac{s}{s+50} imes rac{s}{s+500}$ 

We know that  $C_1 > C_2 > C_3$ . So as we move from low frequency to high frequency,  $C_1$  will short before  $C_2$ , which will short before  $C_3$ .

From the transfer function, we find the frequency of the three poles:

$$egin{aligned} \omega_{p1} &= 5rac{rad}{sec}, \qquad \omega_{p2} &= 50rac{rad}{sec}, \qquad w_{p3} &= 500rac{rad}{sec}\ au_{p1} &= rac{1}{5}sec, \qquad au_{p2} &= rac{1}{50}sec, \qquad au_{p3} &= rac{1}{500}sec \end{aligned}$$

Let's start from the low frequency response and move up. Using the *short circuit* test we look at which capacitor shorts first by initially assuming all the capacitors act as open circuits.



Let  $C_1$  have the pole frequency at 5. Then opening all other capacitors. The short circuit time constant for  $C_1$  is as follows. Followed by the computation for the capacitance

$$au_{sc}^{C_1} = (1000\Omega + 1000\Omega)(C_1) = au_{p1} = rac{1}{5}$$
 $C_1 = rac{1}{10000} = \boxed{100\mu\mathrm{F}}$ 

Now consider  $C_1$  shorted, we continue to move up the frequency. Let  $C_2$  have the pole frequency at 50. We open the remaining  $C_3$  capacitor.



Figure 1.2: Low frequency short circuit test on  $C_2$ 

The short circuit time constant for  $C_2$  is as follows and its corresponding capacitance can be computed.

$$au_{sc}^{C_2} = ((1 \mathrm{k}\Omega \parallel 1 \mathrm{k}\Omega) + 1 \mathrm{k}\Omega)(C_2) = au_{p2} = rac{1}{50} 
onumber \ C_2 = rac{1}{(1500)(50)} = \boxed{13.33 \mu \mathrm{F}}$$

Now consider every capacitor shorted, we finally look at  $C_3$  and let its pole frequency to be 500.



Figure 1.3: Low frequency short circuit test on  $C_3$ 

The short circuit time constant and the capacitance is

$$egin{aligned} & au_{sc}^{C_3} = ((1 \mathrm{k} \Omega \parallel 1 \mathrm{k} \Omega \parallel 1 \mathrm{k} \Omega) + 1 \mathrm{k} \Omega)(C_3) = au_{p3} = rac{1}{500} \ &C_3 = \boxed{1.5 \mu \mathrm{F}} \end{aligned}$$

#### **AC Simulation**

To run the AC simulation, *CircuitMaker 2000 Student Edition* is used. The following circuit is created on *CircuitMaker*.



Sweeping the AC frequency and then measuring the output (the node after the  $1.5\mu$ F capacitor and before the 1k resistor), we obtain the following bode plot.

#### Magnitude



Phase



Using the sxfer blocks in *CircuitMaker*, the identical response is replicated. The circuit drawn is as follows.



The bode plot of this circuit is identical to the one in the part B.

#### Magnitude

Phase





### Discussion

As one can see, the comparison of the bode plots of the simulated circuit and the circuit of the transfer function block is nearly identical.

## Part II

The circuit that is simulated in this part is drawn in *CircuitMaker* like so:



### Α

Part A is only using the circuit at the very top (with  $1\mu F$  capacitor). The simulation yields the following bode plot.

#### Magnitude



Phase



By analyzing the magnitude bode plot using linear approximation and looking at the slopes, the pole locations are approximated graphically.

Pole	Frequency
$\omega_{p_1}$	10.0 Hz
$\omega_{p_2}$	90.0 Hz
$\omega_{p_3}$	3.16 MHz
$\omega_{p_4}$	40.7 MHz

Table 2.1

#### В

The bottom three circuits from the circuit diagram above depicts the variation of circuits used in this section. The circuit is simulated once more and the bode plot is revealed.



The bandwidth of this band-pass filter increases as its low 3 dB cutoff frequency decreases when we increase the capacitance of  $C_3$ .

First, we shall compute the 3 dB frequencies by hand for each  $C_3$  variation.

#### **Calculated Frequencies**

#### **High Frequency Response**

First let's look at the high-frequency response. Since the capacitor we are changing on affect the low frequency response, all the variations should have the same high frequency 3 dB cutoff frequency.

To compute  $\omega_{H3dB}$ , first we short the low frequency capacitors. This is the circuit we see at high frequency:



Let's take a guess that  $C_4$  will short before  $C_2$  and we proceed to the short circuit time constant test.

$$au_{sc}^{C_4} = ((50 \parallel 1000 + 1000) \parallel 1000)(100 \mathrm{pF}) = 5.116 imes 10^{-8} \mathrm{s} \ \omega_{Hp_1} = 19.455 \mathrm{Mrad/s}$$

Now let's observe  $C_2$  with  $C_4$  shorted.

$$au_{sc}^{C_2} = (50 \parallel 1000 \parallel 1000)(100 \mathrm{pF}) = 4.545 imes 10^{-9} \mathrm{s} \ \omega_{Hp_2} = 220 \mathrm{Mrad/s}$$

Indeed the calculated frequency for when  $C_2$  and  $C_4$  match our assumption and makes sense. Let's proceed to compute the value for the high frequency 3 dB frequency.

$$egin{aligned} au_{H3dB} &= \sqrt{ au_{Hp_1}^2 + au_{Hp_2}^2} = 5.316 imes 10^{-8} \mathrm{s} \ \omega_{H3dB} &= 19.47 \mathrm{Mrad/s} = egin{aligned} 3.10 \mathrm{MHz} \ \end{array} \end{aligned}$$

#### Low Frequency Response

We can find the low frequency poles by treating the high capacitors as open circuits.

The  $C_1$  ( $20\mu$ F) will short first so we treat  $C_3$  as open circuit. Our circuit will look like



The short circuit time constants, and subsequently the pole frequency, for this capacitor is

$$egin{aligned} & au_{sc}^{C_1} = (1050)(20\mu) = 21 imes 10^{-3} \mathrm{s} \ & \omega_{Lp_1} = 47.619 \mathrm{rad/s} \end{aligned}$$

#### Now we short $C_1$ and look at $C_3$ .

Since we are changing  $C_3$  to experiment, the low 3 dB frequency changes. However, because we are leaving all the other capacitors intact, their pole locations should remain the same. Thus recalculating the location of the poles that doesn't correspond to  $C_3$  would be redundant.

#### For $C_3 = 1\mu F$ :



Following the same algorithm we compute the time constant and pole frequency for  $C_3$ .

$$au_{oc}^{C_3} = ((50 \parallel 1000) + 1000 + 1000)(1\mu) = 2.048 imes 10^{-3} \mathrm{s} \ \omega_{Lp_2} = 488.372 \mathrm{rad/s}$$

Now with the location of the poles known, we compute the approximate location of the low 3 dB frequency.

$$egin{aligned} &\omega_{L3dB} = \sqrt{(47.619)^2 + (488.372)^2} \ &= 490.7 \ \mathrm{rad/s} \ &= \overline{[78.1 \ \mathrm{Hz}]} \end{aligned}$$

For  $C_3 = 2\mu F$ :

As mentioned above, the only re-calculation is the pole of  $C_3$  capacitor.

$$egin{aligned} &\pi^{C_3}_{sc} = (50 \parallel 1000 + 1000 + 1000)(2\mu) = 4.095 imes 10^{-3} \mathrm{s} \ &\omega_{Lp_2} = 244.186 ext{ rad/s} \end{aligned}$$

And then re-calculate the low 3 dB frequency (notice that we did not need to recalculate the other pole because it remained the same).

$$egin{aligned} \omega_{L3dB} &= \sqrt{(47.619)^2 + (244.186)^2} \ &= 248.786 \ \mathrm{rad/s} \ &= \boxed{39.595 \ \mathrm{Hz}} \end{aligned}$$

For  $C_3 = 5\mu F$ :

Same algorithm: recalculate pole of  $C_3$ , then recalculate low 3 dB frequency.

$$au^{C_3}_{sc} = (50 \parallel 1000 + 1000 + 1000)(5\mu) = 10.238 imes 10^{-3} \mathrm{s} \ \omega_{Lp_2} = 97.674 \ \mathrm{rad/s}$$

$$egin{aligned} \omega_{L3dB} &= \sqrt{(47.619)^2 + (97.674)^2} \ &= \boxed{17.294 \, \mathrm{Hz}} \end{aligned}$$

For  $C_3=10\mu\mathrm{F}$ :

$$egin{aligned} & au_{sc}^{C_3} = (50 \parallel 1000 + 1000 + 1000)(10 \mu) = 20.476 imes 10^{-3} \mathrm{s} \ & \omega_{Lp_2} = 48.837 \ \mathrm{rad/s} \ & \omega_{L3dB} = \sqrt{(47.619)^2 + (48.837)^2} \ & = \boxed{10.856 \ \mathrm{Hz}} \end{aligned}$$

#### **Simulated Frequency**

The 3 dB cutoff frequency can be graphically determined in *CircuitMaker* by looking at the intersection of the 3 dB cursor and the bode plot waveform (such as depicted in *figure 2.4*).

First, we shall look at the low frequency 3 dB points. In *table 2.2*, the simulation value is read off the graph. The corresponding calculated value for the frequency is also displayed in the table. Finally, I computed the error of between the calculated and the simulated value using the simulated value as a reference. The error is calculated using the following formula.

	$\%~{ m error} = rac{ { m calculated} - { m simulated} }{{ m simulated}}  imes 100\%$			
	$1 \mu { m F}$	$2\mu { m F}$	$5\mu { m F}$	$10 \mu { m F}$
$\omega_{L3dB}$ (simulated)	83.7 Hz	44.8 Hz	22.3 Hz	15.9 Hz
$\omega_{L3dB}$ (calculated)	78.1 Hz	39.6 Hz	17.3 Hz	10.9 Hz
% Error	6.69 %	11.6 %	22.4 %	31.4 %

Table 2.2

As one can observe, the error increases as  $C_3$  approaches the value of  $C_1$  which will make their poles closer together. The closer the poles are together, the less accurate our methods of calculations are.

Now let's look at the high frequency 3 dB cutoff frequency. Since the capacitor that we changed only affected low frequency response, the high frequency poles are not affected. Indeed in the bode plot (*figure 2.4*), all four variations have the same high frequency 3 dB cutoff frequency at 3.08 MHz. Then, we calculated the error between the simulated frequency and calculated frequency using the simulation as reference.

	$1 \mu { m F}$	$2\mu { m F}$	$5\mu { m F}$	$10 \mu { m F}$
$\omega_{H3dB}$ (simulated)	3.08 MHz	3.08 MHz	3.08 MHz	3.08 MHz
$\omega_{H3dB}$ (calculated)	3.10 MHz	3.10 MHz	3.10 MHz	3.10 MHz
% Error	0.645 %	0.645 %	0.645 %	0.645 %
Table 2.3				

### Part III

The problem in part III has the following circuit (*figure 3.1*) with the *Miller gain*  $\mathbf{k} = -200$ . It is also stated in the problem that we have four poles and the circuit act as a band pass filter.



### Α

#### **Miller Theorem**

Before we continue, we need to deal with the 2pF capacitor that is couples the voltage from the input side and the controlled current source on the right side.

Since we know the miller gain is k = -200, and  $V_2 = kV_1$  where  $V_1$  is the voltage left of the in 2pF reference to ground, and  $V_2$  is the voltage to the right of the 2pF capacitor.

Miller theorem will allow us to separate the 2pF to two separate capacitors. Miller theorem states that

$$Z_1=Z\left(rac{1}{1-k}
ight), \quad Z_2=Z\left(rac{k}{k-1}
ight)$$

Where  $Z_1$  is the left equivalent impedance of the 2pF capacitor,  $Z_2$  is the right equivalent impedance, and Z is the impedance of the 2pF capacitor.

Since the impedance of a capacitor is the inverse of its capacitance, we can rewrite Miller's theorem to express our interest in the capacitance (by just inversing the factor that multiplies the impedance).

$$C_{\mu1}=C_{\mu}\left(1-k
ight), \quad C_{\mu2}=C_{\mu}\left(rac{k-1}{k}
ight)$$

Where  $C_{\mu 1}$  is the equivalent capacitance added in parallel to the left side,  $C_{\mu 2}$  is the equivalent capacitance added in parallel to the right side, and  $C_{\mu}$  is the 2pF capacitor that is across the two sides.

#### **Miller Equivalence**

$$egin{aligned} C_{\mu 1} &= 2 \ {
m pF}(1-(-200)) \ &= 2 \ {
m pF}(201) \ &= 402 \ {
m pF} \ \end{aligned}$$
 $C_{\mu 2} &= 2 \ {
m pF}\left(rac{-200}{-200-1}
ight) \ &= 2 \ {
m pF}(0.995) \end{aligned}$ 

$$= 1.995 \text{ pF}$$
  
 $C_{\mu 1} \approx 2 \text{ pF}$ 

We will approximate  $C_{\mu 2}$  to just 2pF for the sake of simplicity. After doing Miller equivalence, our new circuit is as depicted in *figure 3.2*.



Finally, we can simplify even further as capacitors in parallel add up. The 400pF and 10pF capacitor on the left side add together to form a single 412pF. Now our updated circuit is displayed in *figure 3.3*. Also notice that we have four capacitors, just what we need for the four poles of the bandpass filter!



#### **Midband Gain**

At midband, the high frequency capacitors remain open circuit, and the low frequency capacitors remain short circuit. The resulting circuit is as follows (*figure 3.4*)



Now that the left side and the right side circuits are decoupled, we can solve each side separately. For the sake of not shooting ourselves in the foot, we shall solve the left side first. The left side is a voltage divider.

$$egin{aligned} V_1 &= V_i \left( rac{1000}{50+1000} 
ight) \ &= 0.952 V_i \end{aligned}$$

Next, the output voltage of the right side is

$$V_o = (-0.1V_1)(4000 \parallel 4000)$$

We use  $V_1$  solved from the left side and substitute it into the equation. Then we can rearrange to find the gain.

$$V_o = (-0.1)(0.952)(V_i)(2000)$$
  
 $\frac{V_o}{V_i} =$  -190.476

#### **Pole Frequencies**

We employ the *short circuit time constant test* to find the pole location of input and output side.

The input side where we will conduct the test is shown in *figure 3.5*, and the output side is shown in *figure 3.6*.

**SCTC Test on Input** 



First we open all the capacitors and see which one shorts first. By intuition, since  $C_1$ , the  $1\mu$ F capacitor is very large, I suspect that this one will short first.

$$egin{aligned} & au_{sc}^{C_1} = (1\mu\mathrm{F})(50+1000) \ &= 1.05~\mathrm{ms} \end{aligned}$$

Now this  $1\mu$ F capacitor is shorted, we continue to do the test on  $C_2$ , the 412pF capacitor.

$$egin{aligned} & au_{sc}^{C_2} = (412 \mathrm{pF})(50 \parallel 1000) \ & = 196.2 \ \mu \mathrm{s} \end{aligned}$$

**SCTC Test on Output** 



The dependent source, as far as the output is concerned, can be treated as an independent source

With that in mind, we can begin the *short circuit time constant test*. First, we shall open all the capacitors. And because  $C_4$ , the bigger 1 $\mu$ F capacitor is larger, it is a relatively low frequency capacitor, and thus will short first.

$$au_{sc}^{C_4} = (1 \mu {
m F})(4000 + 4000) \ = 8.0 \ {
m ms}$$

Now the  $C_4$  capacitor is shorted, and we look at  $C_3$ , the 2pF capacitor.

$$au_{sc}^{C_3} = (2 \ \mathrm{pF})(4000 \parallel 4000) \ = 4.0 \ \mathrm{ns}$$

Now that we know the time constants for all the capacitors, we can compute the pole frequencies.

$$egin{aligned} &\omega_{Lp_1} = rac{1}{8.0 \ {
m ms}} = 125 \ {
m rad/s} o f_{Lp_1} = \ egin{bmatrix} 19.9 \ {
m Hz} \ 19.9 \ {
m Hz} \ \end{pmatrix} \ &\omega_{Lp_2} = rac{1}{1.05 \ {
m ms}} = 952 \ {
m rad/s} o f_{Lp_2} = \ egin{bmatrix} 152 \ {
m Hz} \ 100 \ {
m Hz} \ \end{pmatrix} \ &\omega_{Hp_1} = rac{1}{196.2 \ \mu {
m s}} = 51.0 \ {
m Mrad/s} o f_{Hp_1} = \ egin{bmatrix} 8.11 \ {
m MHz} \ 100 \ {
m MHz} \ \end{pmatrix} \ &\omega_{Hp_1} = rac{1}{4.0 \ {
m ns}} = 250 \ {
m Mrad/s} o f_{Hp_1} = \ egin{bmatrix} 39.8 \ {
m MHz} \ \end{array} \end{aligned}$$

В

The circuits shown in *figure 3.1* and *figure 3.3* are both simulated. The magnitude and phase bode plot is as follows (*figure 3.7* and *figure 3.8*).



Notice the error of the bode plot at high frequencies for the circuits before Miller and after Miller is very large.

The magnitude bode plot with higher high frequency gain is the plot for the circuit before Miller equivalence, and the one below with straighter lines is the circuit after Miller.

The phase bode plot with the high frequency phase shift of 0<sup>o</sup> is **after** Miller, and the one with -90<sup>o</sup> is the original circuit (**before** Miller).

For the sake of simplicity (and due to the method taught in class), we will continue with the circuit after Miller equivalence.

#### **Pole Location Identification**

*Linear Approximation* is the method I decided to use to approximate where the poles are graphically.

Using the graphics program *Adobe Photoshop CC* (screenshot in *figure 3.9*), we can draw lines with exactly -40, -20, 0, 20, and 40 db/decade lines. We assembly the lines tangent to the bode plot such that the linear plot (made using line segments) would resemble the actual plot as close as possible. The constructed linear plot is displayed in \*figure 3.10.



Figure 3.9: Using Adobe Photoshop CC as a tool to linear approximate the poles in the bode plot



Next, we find the intersection of the linear lines, and record their frequency (as seen in *figure 3.11*).



Using the linear approximation method described above, we obtain the four pole values (in *table 3.1*). We also compare the simulated value to the calculated value from the previous section. This comparison is also outlined in *table 3.1*.

Poles	Simulated Frequency	Calculated Frequency	% Error
$\omega_{Lp_1}$	17.9 Hz	19.9 Hz	11.2 %
$\omega_{Lp_2}$	155 Hz	152 Hz	1.94 %
$\omega_{Hp_1}$	7.84 MHz	8.11 MHz	3.44 %
$\omega_{Hp_2}$	39.8 MHz	39.8 MHz	0.025 %

Table 3.1

There is some inaccuracy when it comes to finding the pole location graphically, since the thickness of the pen we draw with has limits. Hence the mild discrepancies of the error.

#### **3 dB Frequencies**

#### Calculated 3 dB Frequencies

Before we go and look at the simulated plots, we should compute the theoretical value of the 3 dB cutoff frequencies.

The low frequency and high frequency poles from previous part can be found in *part A* or in *table 3.1* above. Using these pole frequencies, we can compute the approximate value of the 3 dB frequencies.

$$egin{aligned} &\omega_{H3dB} = \sqrt{\omega_{Lp_1}^2 + \omega_{Lp_2}^2} \ &= \sqrt{(19.9)^2 + (152)^2} \ &= \boxed{153~\mathrm{Hz}} \end{aligned}$$

$$egin{aligned} & au_{H3dB} = \sqrt{ au_{Hp_1}^2 + au_{Hp_2}^2} \ & = \sqrt{\left(rac{1}{8.11 ext{ MHz}}
ight)^2 + \left(rac{1}{39.8 ext{ MHz}}
ight)^2} \ & = 1.25 imes 10^{-7} ext{ s} \ & \Rightarrow \ & \omega_{H3dB} = \left[ 7.95 ext{ MHz} 
ight] \end{aligned}$$

#### **Simulated 3 dB Frequencies**

Using the cursor feature in *CircuitMaker* such as shown in *figure 3.7*, we find the 3 dB to be as follows.

$$\omega_{L3dB} = \boxed{151.2 ext{ Hz}}$$
 $\omega_{H3dB} = \boxed{7.897 ext{ MHz}}$ 

#### Comparison

This table outlines the comparison of the calculated and simulated 3 dB cutoff frequencies and their corresponding errors. The errors are calculated using the simulated value as reference.

	Simulated Frequency	Calculated Frequency	Error
$\omega_{L3dB}$	151.2 Hz	153 Hz	1.19 %
$\omega_{H3dB}$	7.90 MHz	7.95 MHz	0.646 %

Table 3.2

We can see that the calculation is very close to the simulated frequency.

## 3. Conclusion

In *Part I*, we explored method of open circuit/short circuit time constant tests to help us choose capacitor values in order to a specific requirement. In this case, the requirement is the transfer function given by the problem. We also experimented with using transfer function blocks as equivalent circuits. Through simulation, we verified the accuracy of our design choice.

In *Part II*, we determined the low and high 3 dB frequency. We saw the effects of changing capacitor values in order to yield a larger bandwidth. Additionally, we observed that as one of the capacitor under test is close to another capacitor, the calculated frequency is more erroneous compared to the simulated value. This is due to the poles moving too close together, thus some of our methods (which depends on approximation) is no longer as accurate as before.

Finally, in *Part III*, we employed the linear approximation tactic to find the pole locations graphically. The calculated frequency error compared to the simulated frequency is adequate.

Overall, we acquired a set of useful tools and techniques that will assist us in future circuit design and analysis.

## 4. Reference

- 1. ELEC 301 Course Notes
- 2. A. Sedra and K.Smith, "Microelectronic Circuits", 5<sup>th</sup> (or higher) Ed., Oxford University Press, New York.
- 3. CircuitMaker<sup>TM</sup> User's Manual
- 4. PSIM User's Manual
- 5. Notes on ELEC 301 Connect