# Problem Set 7 (Again)

November 26, 2017 17:41

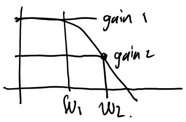
Q1. $A = 10^4$, $\beta = 10^{-2} = 0.01$ Actual gain $A_f = 7\times10^3$

a. $\frac{7\times10^3}{10\times10^3} = 70\%$ of the intended gain

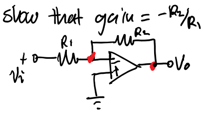
b. $A_f = \frac{A}{1+A\beta}$

$$\frac{dA_f}{dA} = \frac{d}{dA}\left(A(1+A\beta)^{-1}\right)$$
$$= (1+A\beta)^{-1} + A\cdot\left(-(1+A\beta)^{-2}\cdot\beta\right)$$
$$= \frac{1}{1+A\beta} - \frac{A\beta}{(1+A\beta)^2}$$
$$= \frac{1+\cancel{A\beta}-\cancel{A\beta}}{(1+A\beta)^2}$$
$$= \frac{1}{(1+A\beta)^2}$$

c.

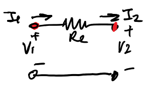



Q2. show that gain $= -R_2/R_1$




Voltage amplifier,

Feedback network

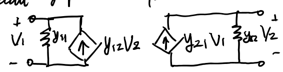



Shunt-shunt topology → y-parameters
↳ current controlled voltage source

Recall y-params

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Recall y-param equivalent circuit:




Finding y-parameters:

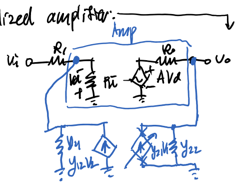
$$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{1}{R_2}$$

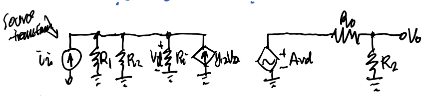
$$\beta = y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = -\frac{1}{R_2}$$

(not important) $y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{1}{R_2}$

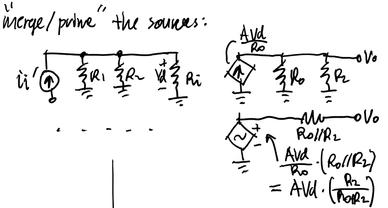
$$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{1}{R_2}$$

Idealized amplifier:

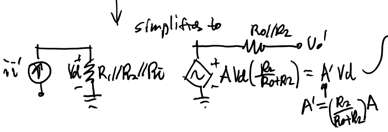



"merge / prune" the sources:




simplifies to




open loop gain $= \frac{V_o'}{i_i'} = \frac{A'(\cancel{i_i'}(R_1//R_2//R_i))}{\cancel{i_i'}}$

$= A'(R_1//R_2//R_i)$

$A''$ ← open loop gain.

From the idealized amplifier, closed loop gain is $A_f = \frac{V_o}{i_i}$, where $i_i = V_i \cdot R_1$

$$A_f = \frac{V_o}{\left(\frac{V_i}{R_1}\right)} = R_1 \cdot \frac{V_o}{V_i}$$

But $A_f$ is the gain with units $V/A$, we want the voltage gain: $\frac{V_o}{V_i}$.

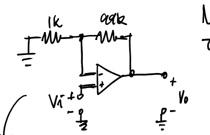
Thus Voltage gain $A_v = \frac{V_o}{V_i}$. $V_i = i_i R_1$ → $A_v = \frac{V_o}{i_i R_1}$

Notice that $A_f = \frac{V_o}{i_i}$, but if the open loop gain $A''$ is large, $A_f = \frac{V_o}{i_i} \to \frac{1}{\beta}$.

And $\frac{1}{\beta} = \frac{1}{\left(-\frac{1}{R_2}\right)} = -R_2$.

Thus $A_v = \frac{V_o}{i_i R_1} = \frac{A_f}{R_1} = \boxed{\frac{-R_2}{R_1}}$

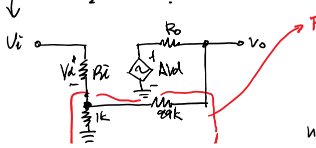
Q3.



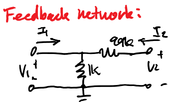

Non-inverting amp: $A = 10^5$ V/V, BW = 10Hz

Topology: Voltage-series (or series-shunt) ⇒ use h-params.

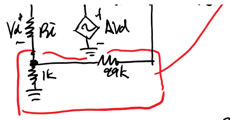
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

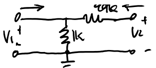



Feedback network:




$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} = 1k // 99k = 990$





$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} = 1k \,//\, 99k = 990$$

$$\beta = h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = \frac{1k}{1k+99k} = 0.010$$

$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0} = / \quad \text{(not important)}$$

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} = \frac{1}{100k} = 1\times10^{-5}$$

With feedback:

gain:

$$A_f = \frac{A}{1+A\beta} = \frac{10^5}{1+10^5(0.010)} = 99.9 \ V/V$$

BW: $BW_f = BW(1+A\beta) = 10.01\text{kHz}$

If $A = 5\times10^4$?

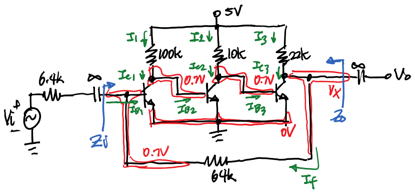
gain:

$$A_f = \frac{A}{1+A\beta} = \frac{5\times10^4}{1+5\times10^4(0.01)} = 99.8 \ V/V$$

BW:

$$BW_f = BW(1+A\beta) = 5.01\text{kHz}$$

## Q4. Find $A_M$, $Z_i$, $Z_o$




If $V_x > 0.7V$, then $I_3 = \frac{5-V_x}{22k} < \frac{1}{2.2}\underbrace{\left(\frac{5-0.7}{10k}\right)}_{I_2}$

Given default $h_{FE} = 100$, it follows that $I_{B3} <$ ½% of $I_2$

Thus we can make the assumption that $I_2 \approx I_{C2}$.

$\Rightarrow I_{C2} \approx \frac{5-0.7V}{10k} = 430\mu A$

$I_{B2} = 4.3\mu A$

$I_{C1} = I_1 - I_{B2} = \frac{5-0.7}{100k} - 4.3\mu A = 38.7\mu A$

$I_{B1} = 0.387\mu A$

Capacitor decoupling,

$\Rightarrow I_{B1} = I_f$

Voltage across 64k $= 64k \cdot 0.387\mu A = 0.025V$

$\Rightarrow V_x = 0.7V +$ voltage across the 64k.

$= 0.725V$

Finally calculate missing pieces,

$\Rightarrow I_3 = I_{C3} = \frac{5-0.725V}{22k} = 194.33\mu A$

$I_{B3} = 1.943\mu A$ (Less than ½% of $I_2$, as expected)

Calculating BJT parameters:

$g_{m1} = \frac{I_{C1}}{V_T} = 0.00155 \mho$

$g_{m2} = 0.0172 \mho$

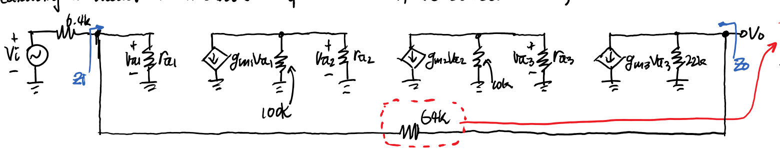
$g_{m3} = 0.00777 \mho$

$r_{\pi 1} = 64.6k$

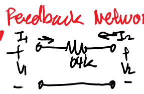
$r_{\pi 2} = 5.81k$

$r_{\pi 3} = 12.865k$

Redrawing the model at midband (∞ capacitors shorted, DC sources shorted)




Feedback Network:




Feedback is sampling voltage (shunt) and gives current (shunt) to the input, thus the topology is shunt-shunt; use $\boxed{\text{Y-parameters}}$.

Y-parameters:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{1}{64k}$

$y_{21} = \ldots\ldots$

$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = -\frac{1}{64k} = \beta$

$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{1}{64k}$

Simplifying the circuit:

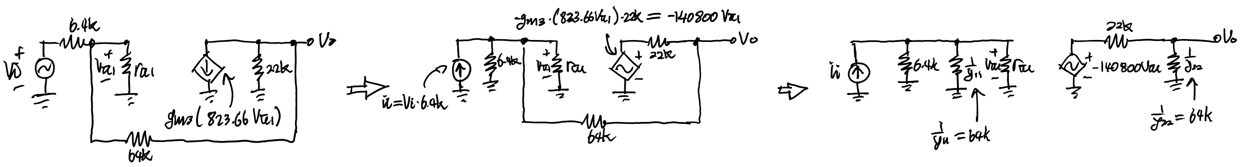
$V_{\pi 2} = -g_{m1}V_{\pi 1} \cdot (100k \,//\, r_{\pi 2})$

$= -8.51 V_{\pi 1}$

$V_{\pi 3} = -g_{m2}V_{\pi 2} \cdot (10k \,//\, r_{\pi 3})$

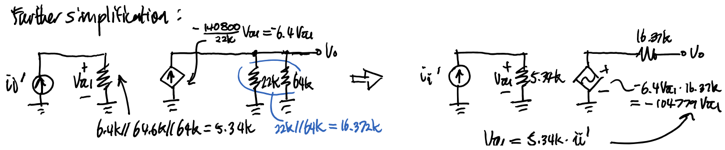
$= -96.776 V_{\pi 2}$

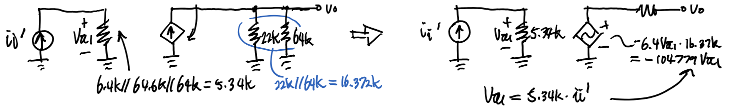
$= 823.66 V_{\pi 1}$




Because of the chosen shunt-shunt topology, which is current controlled voltage source, we make the input a current source, and output a voltage source

Further simplification:






$V_{\pi 1} = 5.34k \cdot i_i'$

$\rightarrow -104779 V_{\pi 1} = -559.5\times10^{6} V_{\pi 1}$

$\Rightarrow A = -559.5\times10^{6}$

$$A_f = \frac{A}{1+A\beta} = \frac{-559.5\times10^{6}}{1+\underbrace{(-559.5\times10^{6})(-1.563\times10^{-5})}_{A\beta = 8745}}$$

$\beta = y_{12} = -1.563\times10^{-5}$

$A_f = -63.99\,k\Omega = \frac{V_o}{i_i}$

$A_m = \frac{V_o}{V_i}$, but $V_i = i_i \cdot 6.4k$, $A_m = \frac{V_o}{i_i \cdot 6.4k}$, $\frac{V_o}{i_i} = -63.99\,k\Omega$

$\rightarrow A_m = \frac{-63.99\,k\Omega}{6.4k} = \boxed{-9.999}$

Input Impedance:

$R_i' = 6.4k // 64.6k // 64k = 5.34k$ (with source impedance)

$R_{if} = R_i' \frac{1}{1+A\beta} = 5.34k\left(\frac{1}{1+8745}\right) = 0.611\Omega = R_i // 6.4k$ (since we want the part w/o source impedance)

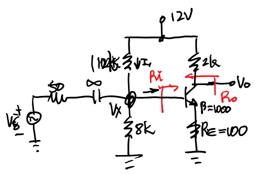
$\Rightarrow R_i = \boxed{0.611\Omega}$

Output impedance

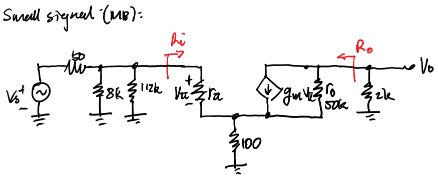
$R_o' = 22k // 64k = 16.372K$

$R_{of} = \frac{R_o'}{1+A\beta} = \boxed{1.872\Omega}$

Q5. Use series-series - topology → z-parameter → Voltage controlled current source




Small signal (MB):




DC operating point:

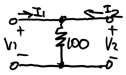
$\frac{12-V_x}{112k} = \frac{V_x}{8k} + I_B$ , $\frac{V_x - 0.7}{100} = (1+\beta)I_B$ → $V_x = 0.793V$, $I_B = 0.93\mu A$

$I_c = 0.93mA$

$g_m = 0.037$

$r_\pi = 26.89k\Omega$

Feedback Network:




$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 100$

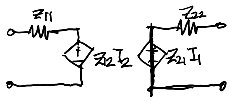
$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 100 = \beta$

$z_{21}$ ——

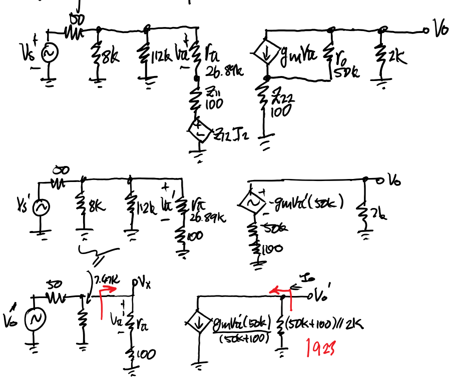
$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 100$

z-params:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$




Small signal with z-param equivalent circuit:




$A_v = \frac{V_o}{V_s}$

$V_o = -g_m V_\pi'\left(\frac{50k}{50k+100}\right) \cdot ((50k+100)//2k)$

$= -71.1401\, V_\pi'$

$V_\pi' = V_x\left(\frac{100}{100+r_\pi}\right) = 0.004 V_x$

$V_o = -0.265 V_x$

$V_x = \frac{(r_\pi+100)//7.47k}{(r_\pi+100)//7.47k + 50} V_s$

$= 0.992 V_s$

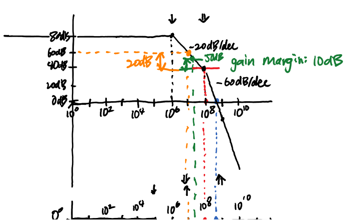
$V_o = -0.262 V_s$

$A_v = \frac{V_o}{V_s} = -0.262$

Q6. OLTF: $T(s) = \frac{10^4}{\left(1+\frac{s}{10^6}\right)\left(1+\frac{s}{10^8}\right)^2} = \frac{10^4 \cdot 10^6 \cdot 10^8 \cdot 10^8}{(s+10^6)(s+10^8)(s+10^8)} = \frac{10^{26}}{(s+10^6)(s+10^8)^2}$ ($k_{DC}$)

Bode Plot:

$|T(0)| = 20\log_{10}(10^4) = 80dB$



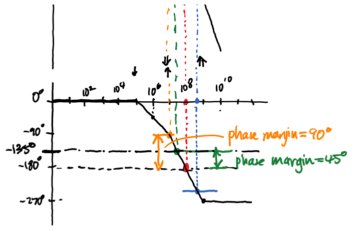

b) want $\beta$ such that ph-margin = 45° (angle = -135°)

at the frequency at which $\theta = -135°$, magnitude is 50dB,

$\Rightarrow 20\log\left(\frac{1}{\beta}\right) = 50dB$

$\boxed{\beta = 3.162\times10^{-3}}$

c). 10 dB




$\Rightarrow 20\log(\frac{1}{\beta}) = 50dB$

$\boxed{\beta = 3.162\times10^{-3}}$

c). 10 dB

d). $20\log(\frac{1}{\beta}) = 40dB$, $\beta = \frac{1}{10^2} = \boxed{0.01}$

e). When $\beta = 10^{-3}$, $20\log(\frac{1}{\beta}) = 60dB$

gain margin = 20dB

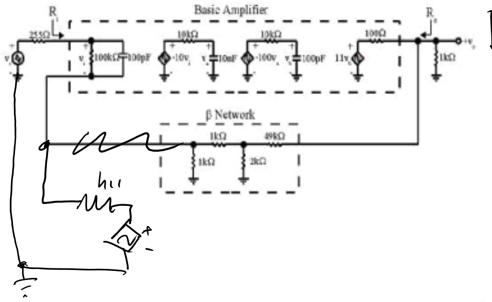
phase margin = 90°

## Q7.

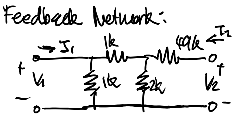
7. For the circuit shown in figure 5 use feedback techniques to find the mid band gain, $A_M = v_o/v_s$, the gain margin, G.M., the phase margin, P.M. = $\varphi_1 - \varphi_{180}$, the input resistance, $R_{in}$ of the amplifier as shown, and output resistance, $R_o$, of the amplifier as shown. Use the series-shunt feedback topology.

→ h-parameters

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$




Feedback Network:



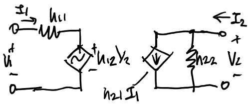

$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} = ((49k // 2k)+1k)//1k = 745\Omega$

$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = \left(\frac{1k}{1k+49k}\right)\cdot 0.5 = 0.010\ V/V$

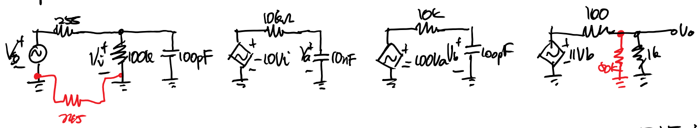
$h_{21}$ = don't care

$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} = \frac{1}{(2k//2k)+49k} = \frac{1}{50k} = 2\times10^{-5}\ ℧$

Equivalent circuit of the feedback network




Ideal Amplifier:




$R_i = 255+745+100k = 101k$

$R_{if} = R_i(1+A\beta)$

$R_{in} = R_{if} - 255$

Embedding everything: $V_s \to V_s'$, $V_o \to V_o'$

$$A_M' = \frac{V_o'}{V_i'} = \frac{V_o'}{V_b}\cdot\frac{V_b}{V_a}\cdot\frac{V_a}{V_i'}\cdot\frac{V_i'}{V_s}$$

$50k//1k = 980$

$\frac{V_o'}{V_b} = 11\cdot\left(\frac{980}{980+100}\right) = 9.982$

$\frac{V_b}{V_a} = -100$ (high-freq capacitor acts open)

$\frac{V_a}{V_i'} = -10$ (high freq capacitor acts open)

$\frac{V_i'}{V_s} = \frac{100k}{100k+(255+745)} = 0.99$

$A_M' = 9883.021\ V/V$

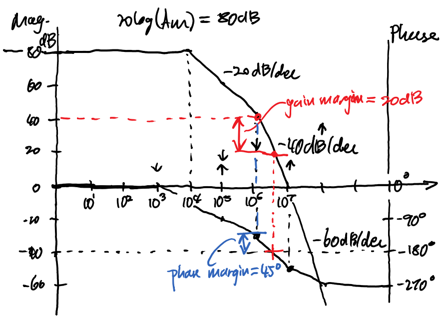
Feedback Gain: $A_f = \frac{A'}{1+A'\beta} = \frac{9883}{1+9883(0.01)} = \boxed{98.978\ V/V}$

For Bode plots, need to find pole locations:

$\omega_{p1} = \frac{1}{1k\cdot100pF} = 10^7\ rad/s$

$\omega_{p2} = \frac{1}{10k\cdot10nF} = 10^4\ rad/s$

$\omega_{p3} = \frac{1}{10k\cdot100pF} = 10^6\ rad/s$




$20\log(\frac{1}{\beta}) = 40dB$

$$R_{of} = \frac{50k//1k//100}{1+A\beta} = R_o//1k$$