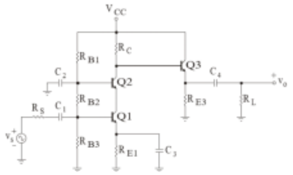
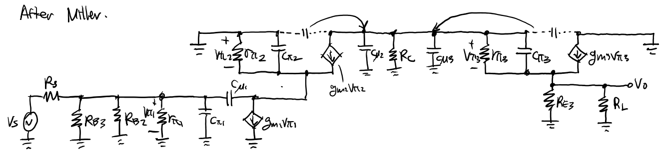
Problem Set 5 (Again)

November 12, 2017 16:26

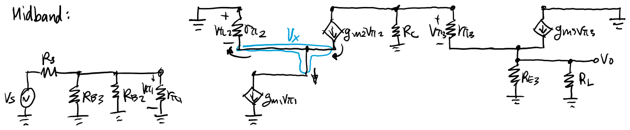
1. For the circuit in Figure 1, do the following:
a) Draw the high-frequency small-signal model.
b) Show that $v_{\pi 1} = v_{\pi 2}$ at midband, irrespective of whether $\beta_1 = \beta_2$ or not.
c) Find expressions for the 3 high-frequency poles.
d) Which of the 3 high-frequency poles do you think will be the dominant pole. Briefly explain your choice.



a. After Miller.



b. Midband:



KCL: $g_{m2}V_{\pi 2} = g_{m1}V_{\pi 1} + \frac{V_x}{r_{\pi 2}}$ $\quad g_m = \frac{I_c}{V_T} = \frac{\beta}{r_\pi}$ $\quad V_x = -V_{\pi 2}$

$$\frac{I_{c2}}{V_T}\cdot V_{\pi 2} = \frac{I_{c1}}{V_T} V_{\pi 1} - \frac{V_{\pi 2}}{r_{\pi 2}}$$

$$\frac{I_{c1}}{V_T}\cdot V_{\pi 1} = \frac{I_{c2}}{V_T}\cdot V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 2}}$$

$$= \frac{\beta_2}{r_{\pi 2}}\cdot V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 2}}$$

$$= V_{\pi 2}\left(\frac{\beta_2+1}{r_{\pi 2}}\right), \quad r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_c}$$

$$= V_{\pi 2}\left(\frac{\beta_2+1}{\left(\frac{\beta V_T}{I_c}\right)}\right)$$

$$\frac{I_{c1}}{\cancel{V_T}}\cdot V_{\pi 1} = V_{\pi 2}\left(\frac{\beta_2+1}{\beta_2}\right)\left(\frac{1}{\cancel{V_T}}\right) I_{c2}$$

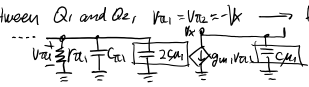
$$I_{c1} V_{\pi 1} = V_{\pi 2}\underbrace{\left(\frac{\beta_2+1}{\beta_2}\cdot I_{c2}\right)}_{I_{E2}}$$

$$\cancel{I_{c1}} V_{\pi 1} = V_{\pi 2} \cancel{I_{E2}}, \quad I_{c1} = I_{E2}$$

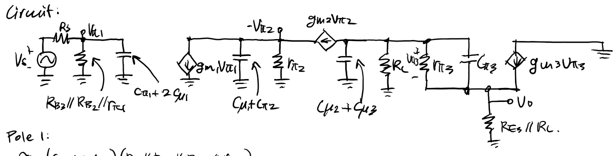
$$\boxed{V_{\pi 1} = V_{\pi 2}}$$

c. Between Q_1 and Q_2, $V_{\pi 1} = V_{\pi 2} = -V_x \longrightarrow K = -1$, Miller $C_\mu' = C_{\mu 1}(1-K) = 2C_{\mu 1}$

$C_{\mu 1}'' = C_{\mu 1}\left(1 - \frac{1}{K}\right) \approx C_{\mu 1}$



Circuit:



Pole 1:
$$\tau_1 = (C_{\pi 1} + 2C_{\mu 1})\underbrace{(R_s // R_{B3} // R_{B2} // r_{\pi 1})}_{< R_s \text{ (small)}}$$

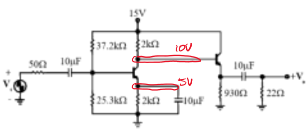
Pole 2:
$$\tau_2 = (C_{\mu 1} + C_{\pi 2})\underbrace{(r_{\pi 2})}_{\text{Kilo ohms range}}$$

Pole 3:
$$\tau_3 = (C_{\pi 3} + C_{\mu 3})\underbrace{(R_c // (r_{\pi 3} + (1+\beta)(R_{E3} // R_L)))}_{\text{Kiloohms}}$$

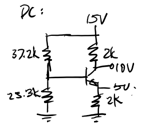
d. Pole 2 and 3 are possibly dominant poles since $\tau_{H3dB} = \sqrt{\tau_{Hp1}^2 + \tau_{Hp2}^2 + \ldots}$ (Bigger → more dominant)

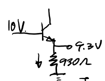
2) Give the transfer function of the circuit shown in figure 2 with numerical values for the poles, zeros and midband gain. You may assume that $V_{C1} = V_{B2} = 10V$ and that $V_{E1} = 5V$. You may also assume that the pole and zero associated with $c_{\pi 2}$ cancel one another. (essentially (C_π doesn't exist)

Default value:
$\beta = 100$, $C_\pi = 10pF$, $C_\mu = 1pF$



DC:



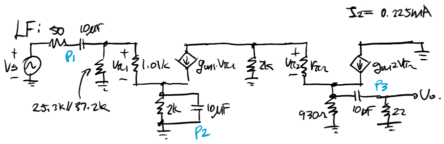


$I_E = 2.5mA$
$I_c = 2.475mA$
$I_1 = 0.25mA$
$V_{B1} = 5.7V$
$I_2 = 0.225mA$

$g_{m1} = \frac{I_c}{V_T} = 0.099℧$
$r_{\pi 1} = 1.01k\Omega$

$I_{E2} = 10mA$
$I_{c2} = 9.9mA$
$g_{m2} = \frac{I_{c2}}{V_T} = 0.396$
$r_{\pi 2} = 252.5\Omega$

LF:



P3: $\omega_{Lp3} = \left[\left(\left(2k + \underset{252.5\Omega}{r_{\pi 2}}\right)\cdot\left(\frac{1}{1+\beta}\right) // 930\right) + 22\right]\cdot 10\mu F\Big]^{-1}$

$= (44\Omega \cdot 10\mu F)^{-1}$

$=$ 2285 rad/s

OCTC/SCTC test for P_1 and P_2:

$$\tau_{oc}^{C_{c1}} = 10\mu F\left(50 + \left(25.3K // 37.7K // ((1+\beta)2k + r_{\pi 1})\right)\right)$$

$= 10\mu F \cdot 14.14k$

$= 0.141 \leftarrow$ lower frequency

$$\tau_{oc}^{CE} = \left(\underbrace{\left((25.3k // 37.2k) + 1.01k\right)\left(\frac{1}{1+\beta}\right)}_{159.} // 2k\right)\cdot 10\mu F$$

$= 1.474\times 10^{-3} s$

$$\tau_{sc}^{CE} = 10\mu F\cdot\left(\left((50 // 23.3k // 37.4k) + 1.01k\right)\left(\frac{1}{1+\beta}\right) // 2k\right)$$

$= 1.044\times 10^{-4} s$

$\omega_{Lp1} = \frac{1}{0.141} =$ 7.092 rad/s

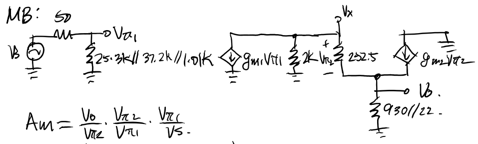
$\omega_{Lp2} =$ 9578.3 rad/s

Zeroes: $\omega_{Lz1} = \omega_{Lz3} = 0$

$\omega_{Lz2} = \frac{1}{2k\cdot 10\mu F} =$ 50 rad/s

$$F_L(s) = \left(\frac{s}{s+9137}\right)\left(\frac{s}{s+7.092}\right)\left(\frac{s+50}{s+9578}\right)$$

MB:



$$A_m = \frac{V_o}{V_{\pi 2}}\cdot\frac{V_{\pi 2}}{V_{\pi 1}}\cdot\frac{V_{\pi 1}}{V_s}.$$

$$\frac{V_{\pi 1}}{V_s} = \frac{(25.3k // 37.2k // 1.01K)}{(25.3k // 37.2k // 1.01K + 50)}$$

$= 0.95$

$V_x = -g_{m1}V_{\pi 1}\left(2k // (252.5 + (1+\beta)(930 // 22))\right)$

$V_x = -108.471 V_{\pi 1} \longrightarrow K = -108.471$

$$V_o = V_x\cdot\frac{(930 // 22)}{(930 // 22) + \left(\frac{1}{1+\beta}\right)(252.5)}$$

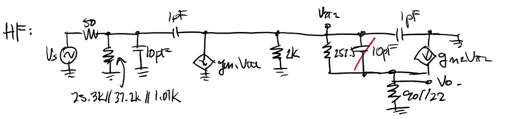
$V_o = 0.896 V_x$

$= -97.168 V_{\pi 1}$

$\frac{V_o}{V_s} = -97.168(0.95)$

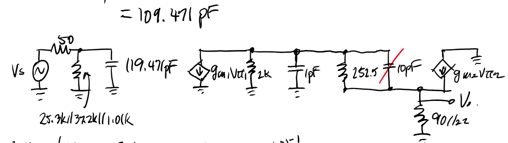
$A_m = \boxed{-92.310}$ V/V.

HF:



Miller: $C_{\mu 1}' = C_{\mu 1}(1-K)$

$= 109.471 pF$

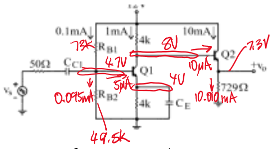


$\omega_{Hp1} = \left(119.471pF\cdot(50 // 25.3k // 37.2k // 1.01k)\right)^{-1}$

$\omega_{Hp2} = \left(1pF\cdot(2k // 252.5 + (1+\beta)(90 // 22))\right)^{-1}$

$$H_f = \left(\frac{\omega_{Hp1}}{s+\omega_{Hp1}}\right)\left(\frac{\omega_{Hp2}}{s+\omega_{Hp2}}\right)$$

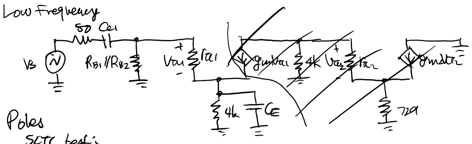
3) The designers of the circuit shown in figure 3 have used a 1/3 rule to bias the amplifier shown. They have also used "pole-zero cancellation" to give the amplifier the low frequency amplitude response of a single time-constant circuit and have put ω_{L3dB} at 1500/s. Assume that $\beta_1 = 200$ and that $\beta_2 = 1000$. What are the values of C_E and C_{C1} ? ($C_E = 26.6\ \mu F$; $C_{C1} = 3.8\ \mu F$)



$r_{\pi 1} = \frac{\beta}{g_m}$, $g_m = \frac{1mA}{25mA} = 0.040℧$

$= 5k$

Low Frequency



Poles

SCTC test:

$\tau_{SC}^{C_{c1}} = C_{c1} \cdot \underbrace{(50 + R_{B1}//R_{B2}//r_{\pi 1})}_{4325.39}$ X

$\tau_{SC}^{C_E} = C_E \cdot \underbrace{\left(\left((50 // R_{B1} // R_{B2}) + r_{\pi 1}\right)\left(\frac{1}{1+\beta}\right) // 4k\right)}_{25}$

$= C_E \cdot 25$ $\rightarrow \omega_{P2} = \frac{1}{25 C_E}$

Zero:

$\omega_{C_{Z1}} = 0$

$\omega_{Z2} = \frac{1}{4k C_E}$

OCTC test:

$\tau_{OC}^{C_{c1}} = C_{c1} \cdot \left(50 + R_{B1}//R_{B2}//(r_{\pi 1} + (1+\beta)(4k))\right)$

$= C_{c1} \cdot 28.51K$

$\rightarrow \omega_{Lp1} = \frac{1}{C_{c1} \cdot 28.51K}$

Pole-zero cancellation ⇒ a pole and zero share the same location:

Since the only pole $\left(\frac{1}{4K \cdot C_E}\right) \neq \left(\frac{1}{25 \cdot C_E}\right)$,

It follows that $\frac{1}{4K C_E} = \frac{1}{C_{C1} \cdot 28.51K}$.

they cancel each other out

Thus leaving us with one pole at $\frac{1}{25 \cdot C_E} = \omega_{L3dB}$

$1500\ rad/s = \frac{1}{25 C_E}$

$\rightarrow \boxed{C_E = 26.67 \mu F}$

Since $4K \cdot C_E = 28.51K\ C_{c1}$

$\rightarrow \boxed{C_{c1} = 3.74 \mu F}$

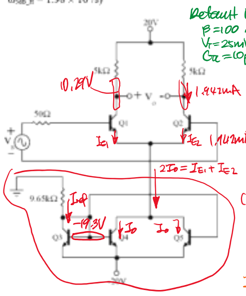
4) Assuming that the differential amplifier shown in figure 4 is attached to a load consisting of a 10 kΩ resistor in parallel with a 100 pF capacitor, find the value of the mid band gain and the high frequency 3 dB point. (A_M = -196 V/V ; $\omega_{3dB\_H} = 1.98 \times 10^7$/s) — HF capacitor load.

Default values:

$\beta = 100$, $V_{BE} = 0.7V$

$V_T = 25mV$ $r_o = \infty$

$C_\pi = 10pF$, $C_\mu = 1pF$



Current Mirror:

$I_{ref} = \frac{0-(19.3V)}{9.65k\Omega}$

$= 2.0mA$

$I_o = \frac{I_{ref}}{1+\frac{3}{\beta}} = \frac{2.0mA}{1+\frac{3}{100}} = 1.942mA$

$2I_o = 3.883mA = I_{E1} + I_{E2}$

Assuming Q1 and Q2 are identical:

$I_{E1} = I_{E2} = 1.942mA$.

$\rightarrow I_{C1} \approx I_{E1} = 1.942mA$

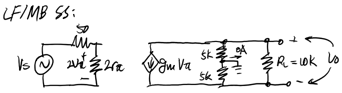
$I_{E2} \approx I_{E2} = 1.942mA$

$V_{C1} = V_{C2} = (20V) - (5K(1.942mA)) = 10.29V$

$g_{m1} = g_{m2} = \frac{I_C}{V_T} = 0.078℧$

$r_{\pi 1} = r_{\pi 2} = 1287.5\Omega$

LF/MB SS:



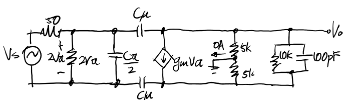
$2V_\pi = \left(\frac{2r_\pi}{2r_\pi + 50}\right) V_s$, $V_\pi = \underbrace{\frac{1}{2}\left(\frac{2r_\pi}{2r_\pi + 50}\right)}_{0.490} V_s$

$V_o = -g_m V_\pi \cdot (10k // 10k)$

$V_o = (-388.35)(0.490) V_s$

$\frac{V_o}{V_s} = A_m = \boxed{-190.5\ V/V}$

HF SS:



Miller: $K = \frac{V_o}{2V_\pi} = \frac{1}{2} \frac{V_o}{V_\pi}$

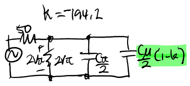
$V_o = -g_m V_\pi \cdot 5k$

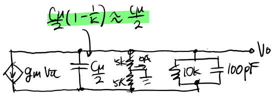
$V_o = -(388.4) V_\pi$

$\frac{V_o}{V_\pi} = -388.4$

$K = -194.2$

$\frac{C_\mu}{2}\left(1 - \frac{1}{K}\right) \approx \frac{C_\mu}{2}$





$\tau_{Hp1} = \left(\frac{C_\pi}{2} + \frac{C_\mu}{2}(1-K)\right)(50 // 2r_\pi)$

$= 5.032 \times 10^{-9} s$

$\tau_{Hp2} = \left(\frac{C_\mu}{2} + 100pF\right)\underbrace{(10k // 10k)}_{5K}$

$= 5.025 \times 10^{-7} s$

$\tau_{H3dB} = 5.025 \times 10^{-7} s$

$\omega_{H3dB} = \boxed{1.99\ Mrad/s}$