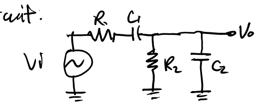
Problem Set 2 (Again)

October 8, 2017 2:56 PM

Q1. Show that sum of the poles == Sum of inverse short circuit time constants.

Circuit.



Short circuit τ Test.

$\tau_{sc}^{C_1} = C_1(R_1 + R_2)$

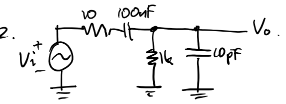
C_1 shorts

$\tau_{sc}^{C_2} = C_2(R_1 \| R_2) = \dfrac{C_2}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$

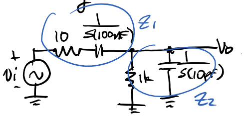
Sum of inverse : $= \dfrac{1}{\tau_{sc}^{C_1}} + \dfrac{1}{\tau_{sc}^{C_2}}$

$= \dfrac{1}{C_1(R_1+R_2)} + \dfrac{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}{C_2}$

Q2.



a. Finding exact transfer function:



$Z_1 = 10 + \frac{1}{s(100n)}$

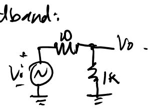
$Z_2 = 1k \| \frac{1}{s(10pF)}$

$\frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$

$= \dfrac{10 \times 10^{9} s}{(s + 10.1 \times 10^{9})(s + 9.9k)}$

b. using OC/SC test.

Midband:



Gain $= \frac{1000}{10+1000}$

$= 0.99$

SC test: (from low→high)

$\tau_{sc}^{C_1} = (10+1000)(100nF) = 0.101\,ms$

$\omega_{P1} = 9.9\,krad/s$

C_1 shorts

$\tau_{sc}^{C_2} = (10 \| 1000)(10pF) = 9.9 \times 10^{-11} s$

$\omega_{P2} = 10.1\,Grad/s$

Template for T(s):

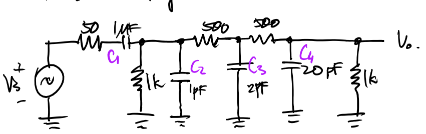
$T(s) = A_m \cdot F_L(s) \cdot F_H(s)$

Low: $\frac{s}{s+\omega_{P1}}$ High: $\frac{\omega_{P2}}{s+\omega_{P2}}$

Thus

$T(s) = (0.99)\left(\dfrac{s}{s+9.9k}\right)\left(\dfrac{10.1G}{s+10.1G}\right)$

Q3. Find 3dB freq.



SC Test:

→ $\tau_{sc}^{C_1} = (50 + (1000 \| (500+500+1000)))(1\mu F)$

$= 0.717\,ms$

$\omega_{LP1} = 1395\,rad/s$

low freq response

C1 shorts

→ $\tau_{sc}^{C_4} = ((50 \| 1000) + 500 + 500) \| 1000) \cdot (20pF)$

$= 1.023 \times 10^{-8} s$

$\omega_{P2} = 97.73\,Mrad/s$

C4 shorts

→ $\tau_{sc}^{C_3} = (((50 \| 1000) + 500) \| 500)(2pF)$

$= 5.227 \times 10^{-10} s$

$\omega_{P3} = 1.913\,Grad/s$

C3 shorts

→ $\tau_{sc}^{C_2} = ((50 \| 1000) \| 500) \cdot 1pF$

high freq response

$\omega_{L3dB} = \sqrt{\omega_{P1}^2 + \omega_{P2}^2 \ldots}$

$= \sqrt{(1395)^2}$

$= \boxed{1395\ rad/s}$

$\tau_{H3dB} = \sqrt{\tau_{HP1}^2 + \tau_{HP2}^2 \ldots}$

$= \sqrt{(1.023 \times 10^{-8})^2 + (5.227 \times 10^{-10})^2 + (4.348 \times 10^{-11})^2}$

$= 1.024 \times 10^{-8} s$

$\omega_{H3dB} = \boxed{97.6\ Mrad/s}$

$\omega_{p2} = 1.913\ \text{Grad/s}$

— C_3 shorts —

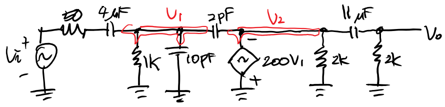
$\rightarrow \tau_{sc}^{C_2} = ((50 // 1000) // 500) \cdot 1\text{pF}$
$= 4.348 \times 10^{-11}\ \text{s}$

$\omega_{p3} = 23\ \text{Grad/s}$

Q4. $\omega_{Lz1} = 100\ \text{rad/s}$, $\omega_{Lz2} = 0\ \text{rad/s}$
$\omega_{Lp1} = 200\ \text{rad/s}$, $\omega_{Lp2} = 50\ \text{rad/s}$

$$\omega_{L3dB} = \sqrt{\omega_{Lp1}^2 + \omega_{Lp2}^2 - 2\omega_{Lz1}^2 - 2\omega_{Lz2}^2}$$
$$= \sqrt{200^2 + 50^2 - 2(100)^2 - 2(0)^2}$$
$$= \boxed{150\ \text{rad/s}}$$

Q5. Use Miller



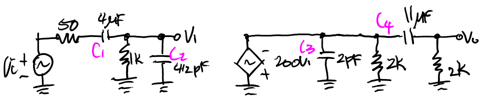
$V_2 = -200 V_1$, thus $k = -200$.

Miller: $Z_1 = Z\left(\frac{1}{1-k}\right)$, $Z_2 = Z\left(\frac{k}{k-1}\right)$

$C_{\mu 1} = C_\mu(1-k)$, $C_{\mu 2} = C_\mu\left(\frac{k-1}{k}\right)$

Then $C_{\mu 1} = 2\text{pF}(1+200)$, $C_{\mu 2} = 2\text{pF}\left(\frac{-200-1}{-200}\right)$
$= 402\text{pF}$ $\approx 2\text{pF}$

And circuit becomes:



We solve midband first:

$V_1 = \frac{1000}{1050} V_i$

$V_o = -200\left(\frac{1000}{1050}\right) V_i$

$A_m = \frac{V_o}{V_i} = -190.476$

Next, low-freq response:

$\tau_{sc}^{C_1} = (50+1000) \cdot 4\mu\text{F}$
$= 0.0042\ \text{s}$
$\omega_{Lp1} = 238.1\ \text{rad/s}$

$\tau_{sc}^{C_4} = (2000)(11\mu\text{F})$
$= 0.022$
$\omega_{Lp2} = 45.455\ \text{rad/s}$

$F_L(s) = \frac{s}{(s+238.1)} \cdot \frac{s}{(s+45.5)}$

Finally, high freq response:

(C_1 and C_4 shorted)

$\tau_{sc}^{C_2} = (50 // 1k) \cdot 412\text{pF}$
$= 1.962 \times 10^{-8}\ \text{s}$
$\omega_{Hp1} = 50.97\ \text{Mrad/s}$

$\tau_{sc}^{C_3} = (0)(2\text{pF})\text{s}$
$\omega_{Hp2} = \infty\ \text{rad/s}$

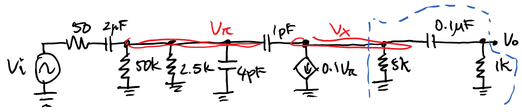
$F_H(s) = \frac{50.97\text{M}}{(s+50.97\text{M})} \cdot \left(\frac{\omega}{s+\omega}\right)$ $\lim_{\omega\to\infty}$

$\lim_{\omega\to\infty} \frac{1}{\left(\frac{s}{\omega}\right)+1} = 1.$

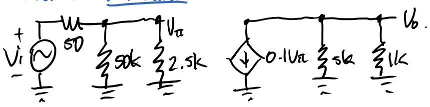
$F_H(s) = \left(\frac{50.97\text{M}}{s+50.9\text{M}}\right)$

$$\boxed{T(s) = -190.476 \cdot \frac{s^2}{(s+238.1)(s+45.5)} \cdot \frac{50.97\text{M}}{s+50.9\text{M}}}$$

Q6.



Look at midband for k



$V_\pi = \frac{50k // 2.5k}{50k//2.5k + 50} V_i$

$V_o = -0.1(5k // 1k) \cdot \frac{(50k//2.5k)}{(50k//2.5k + 50)} V_i$

$\frac{V_o}{V_i} = k = -81.619$

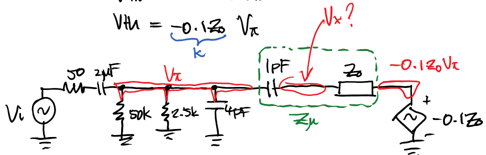
Applying Miller

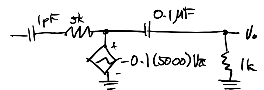
$C_{\mu 1} = 1\text{pF}(1-k)$ $C_{\mu 2} = 1\text{pF}\left(\frac{k+1}{k}\right)$
$= 82.62\text{pF}$ $= 0.988\text{pF}$

THEVENIN: find k

$Z_o = \left(\frac{1}{s \cdot 0.1\mu\text{F}} + 1k\right) // 5k$

$V_{th} = -0.1 V_\pi \cdot Z_o$
$V_{th} = \underbrace{-0.1 Z_o}_{k} V_\pi$





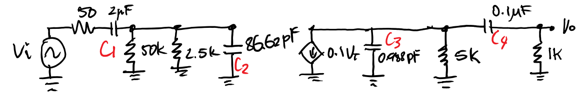
MILLER: separate LHS & RHS

$Z_\mu = \frac{1}{s(1\text{pF})} + Z_o$, $Z_o = \left(\frac{1}{s \cdot 0.1\mu\text{F}} + 1k\right) // 5k$

Applying Miller:

$$C_{\mu 1} = 1\text{pF}(1-K) = 82.62\text{pF} \qquad C_{\mu 2} = 1\text{pF}\left(\frac{K+1}{K}\right) = 0.988\text{pF}$$

a.



b. midband gain $A_m = -81.619$

c. Find all poles.

Low freq:

$$\tau_{sc}^{C_1} = [50 + (50k \parallel 2.5k)] \cdot 2\mu F = 4.862\,ms$$

$$\omega_{Lp1} = 205.681 \text{ rad/s}$$

$$\tau_{sc}^{C_4} = (5k + 1K)(0.1\mu F) = 0.6\,ms$$

$$\omega_{Lp2} = 1666.7 \text{ rad/s}$$

High freq:

$$\tau_{sc}^{C_2} = (50 \parallel 50k \parallel 2.5k) \cdot 86.62pF = 4.242\,ns$$

$$\omega_{Hp1} = 235.742 \text{ Mrad/s}$$

$$\tau_{sc}^{C_3} = (5k \parallel 1k) \cdot 0.988F = 8.233 \times 10^{-10}\,s$$

$$\omega_{Hp2} = 1.215 \text{ Grad/s}$$

d. transfer function:

$$T(s) = A_m \cdot F_L(s) \cdot F_H(s)$$

$$= (-81.619)\left[\left(\frac{s}{s+206}\right)\left(\frac{s}{s+1667}\right)\right]\left[\left(\frac{236\times10^6}{s+236\times10^6}\right)\left(\frac{1.215\times10^9}{s+1.215\times10^9}\right)\right]$$

3dB frequencies:

$$\omega_{L3dB} = \sqrt{\omega_{Lp1}^2 + \omega_{Lp2}^2} = 1679 \text{ rad/s}$$

$$\tau_{H3dB} = \sqrt{\tau_{Hp1}^2 + \tau_{Hp2}^2} = 4.321\,ns$$

$$\omega_{H3dB} = 231.4 \text{ Mrad/s}$$