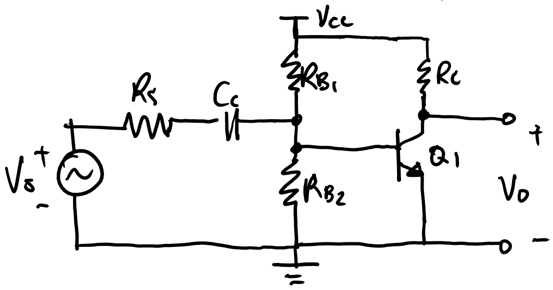
Circuit of Interest

September 7, 2017 3:29 PM

Consider common-emitter amplifier:

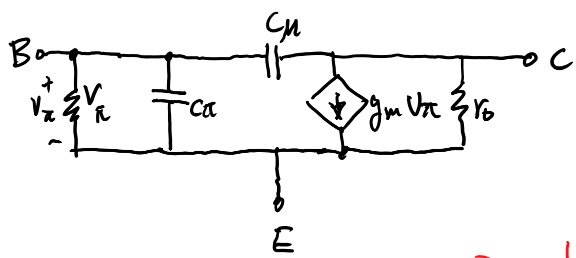


Where:

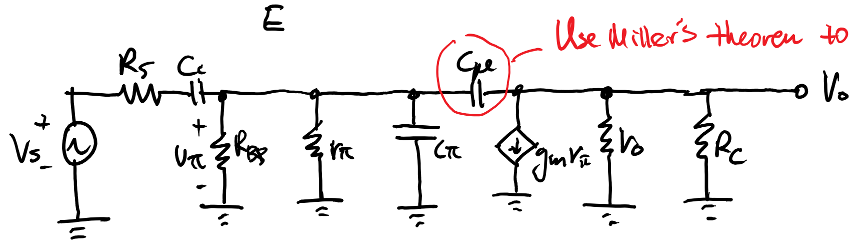
- V_S is source voltage, R_S is the source's internal resistance
- C_c is the coupling capacitor

Using small signal model (→ shorting DC), we represent the circuit using the hybrid π model

BJT:



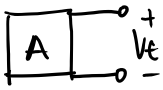
CIRCUIT OF INTEREST:



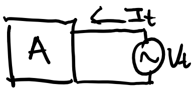
Thevenin, Norton, and Miller

September 23, 2017 11:06 PM

Thevenin:

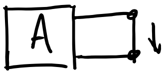


V_t = Thevenin voltage



Z_t (Thevenin impedence) = $\frac{V_T}{I_T}$

Norton

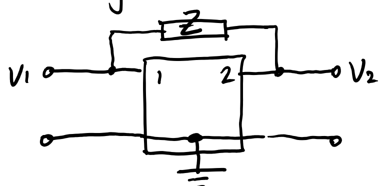


I_n = Norton current

Y_n (Norton admittance) = $\frac{I_t}{V_T}$

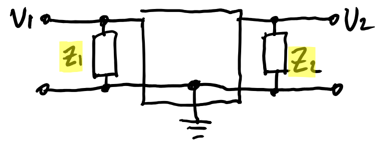
NEW!

Miller: (For given network where $V_2 = kV_1$):



$Z_1 = Z\left(\frac{1}{1-k}\right)$

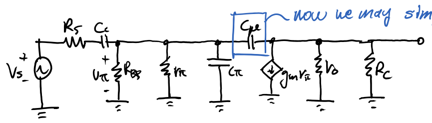
$Z_2 = Z\left(\frac{k}{1-k}\right)$



Circuit of Interest Simplified

September 23, 2017 11:16 PM

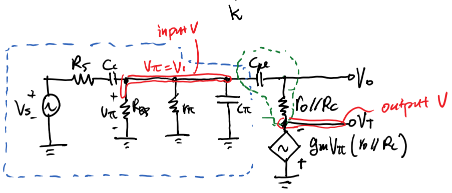
We left the BJT circuit of interest as:



First, we can simplify the RHS:

$$V_T - \underbrace{(r_o \,//\, R_C)}_{Z_{th}}(-g_m V_{\pi})$$

$$\text{Input} \longrightarrow V_T - \underbrace{g_m(r_o \,//\, R_C)}_{k}\, V_{\pi} \longleftarrow \text{Output}$$

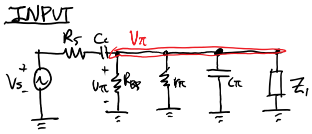


Now let's split C_{μ} in to the $\boxed{\text{input}}$ side and the $\boxed{\text{output}}$ side

Recall that: for $V_2 = kV_1$, where $V_1 = V_{\pi}$, $V_2 = V_T$, $Z = Z_{c\mu} + (r_o // R_C)$, $k = -g_m(r_o // R_C)$

$$Z_1 = Z\left(\frac{1}{1-k}\right), \quad Z_2 = Z\left(\frac{k}{1-k}\right)$$

INPUT

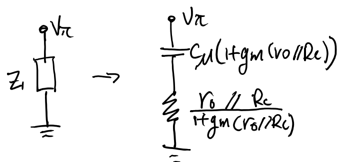


$$Z_1 = \left[Z_{c\mu} + (r_o // R_C)\right]\left(\frac{1}{1+g_m(r_o//R_C)}\right)$$

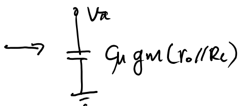
$$= \frac{\left(\frac{1}{j\omega C_{\mu}} + r_o//R_C\right)}{1+g_m(r_o//R_C)}$$

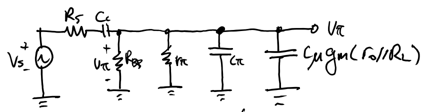
$$= \frac{\left(\frac{1}{j\omega C_{\mu}}\right)}{1+g_m(r_o//R_C)} + \frac{r_o // R_C}{1+g_m(r_o//R_C)}$$

$$= \underbrace{\frac{1}{j\omega C_{\mu}(1+g_m(r_o//R_C))}}_{\text{capacitance}} + \underbrace{\frac{r_o // R_C}{1+g_m(r_o//R_C)}}_{\text{resistance}}$$

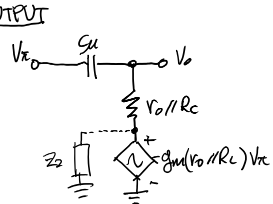


If $\frac{1}{\omega C_{\mu}} \gg r_o // R_C$, and $g_m(r_o//R_C) \gg 1$, then we can further simplify:





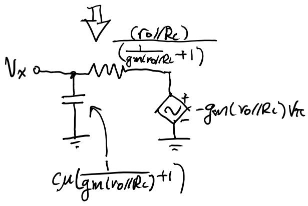
OUTPUT



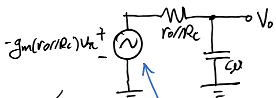
$$Z_2 = \left(\frac{1}{j\omega C_{\mu}} + r_o//R_C\right)\left[\frac{-g_m(r_o//R_C)}{1+g_m(r_o//R_C)}\right]$$

$$= \frac{-g_m(r_o//R_C)}{j\omega C_{\mu}(1+g_m(r_o//R_C))} + \frac{-g_m(r_o//R_C)^2}{1+g_m(r_o//R_C)}$$

$$= \underbrace{\frac{-1\,?}{j\omega C_{\mu}\left(\frac{1}{g_m(r_o//R_C)}+1\right)}}_{\text{new capacitance}} + \frac{-(r_o//R_C)\,?}{\left(\frac{1}{g_m(r_o//R_C)}+1\right)} \quad (-)\,?$$

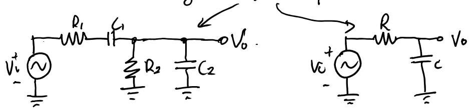


Similar to the input we can approximate given $g_m(r_o//R_C) \gg 1$



As far as output is concerned, this can be treated as an independent source.

So we essentially have 2 simple circuits



Laplace Domain

September 24, 2017 3:08 PM

$$\mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$$

s (complex frequency) $= \sigma + j\omega$

σ ← neper (rate at which func. grows/decays)

$$\mathcal{L}\{Kf(t)\} = K\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{f_1(t) \pm f_2(t)\} = F_1(s) \pm F_2(s)$$

$$\mathcal{L}\{Ke^{at}\} = \frac{k}{s-a}$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-)$$

$$\mathcal{L}\left\{\frac{d^2f(t)}{dt^2}\right\} = s^2F(s) - sf(0^-) - \frac{df}{dt}(0^-)$$

$$\mathcal{L}\left\{\int_0^t f(t)\,dt\right\} = \frac{F(s)}{s}$$

$$\mathcal{L}\{f_1(t) * f_2(t)\} = F_1(s)\cdot F_2(s)$$

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

$$\mathcal{L}\left\{\frac{Kt^{n-1}e^{at}}{(n-1)!}\right\} = \frac{K}{(s-a)^n}$$

$$\mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2+\omega^2}$$

ex. Given transfer function $T(s) = \frac{1000s}{(s+10)(s+1000)}$, and input $V_i(t) = u(t)\sin(10t)$

$$V_o(s) = T(s)\,V_i(s)$$

$$= \underbrace{\left(\frac{s}{s+10}\right)}_{\substack{\text{low freq response} \\ \text{(high pass filter)}}}\underbrace{\left(\frac{1000}{s+1000}\right)}_{\substack{\text{high freq response} \\ \text{(low pass filter)}}} \cdot \left(\frac{10}{s^2+10^2}\right)$$

$$= \frac{10000s}{(s+10)(s+1000)(s^2+100)} = \frac{A}{s+10} + \frac{B}{s+1000} + \frac{Cs+D}{s^2+1000}$$

Setting $s=-10$, mult by $(s+10)$:

$$\rightarrow \frac{10000(-10)}{(1000-10)(200)} = \frac{-50}{99} = A$$

Setting $s=-1000$: mult $(s+1000)$

$$\rightarrow \frac{10000(-1000)}{(10-1000)(1000000+100)} = \frac{10000}{990099} = B$$

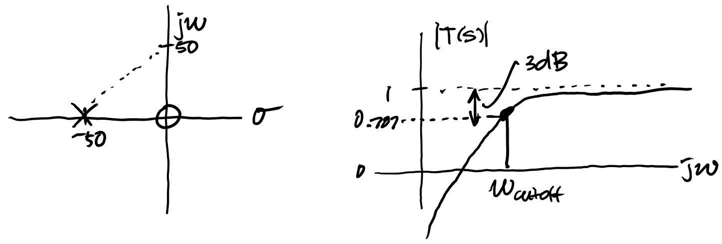
$\vdots$

$$= \underbrace{\frac{10000}{990099}\left(\frac{1}{s+1000}\right) - \frac{50}{99}\left(\frac{1}{s+10}\right)}_{\text{decaying exponentials}} + \underbrace{\frac{4950}{10001}\left(\frac{s}{s^2+100}\right) + \frac{5050}{10001}\left(\frac{1}{s^2+100}\right)}_{\text{sinusoidal}}$$

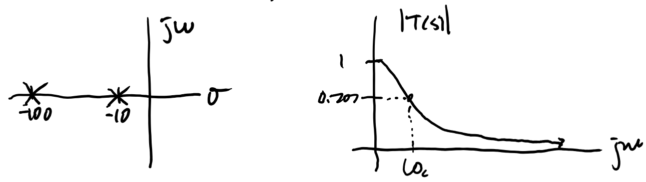
Bode Plots

September 24, 2017 3:34 PM

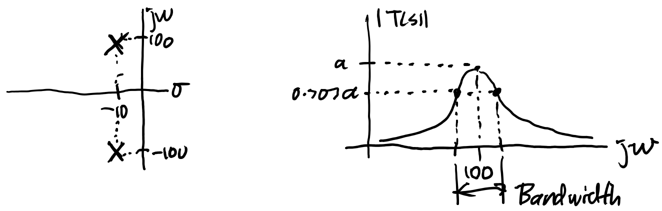
ex. $T(s) = \frac{s}{s+10}$ (zero at zero) (pole at -50)



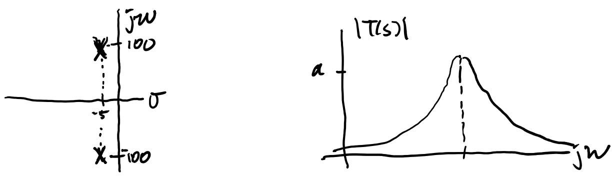
ex. $T(s) = \frac{1000}{(s+10)(s+100)}$



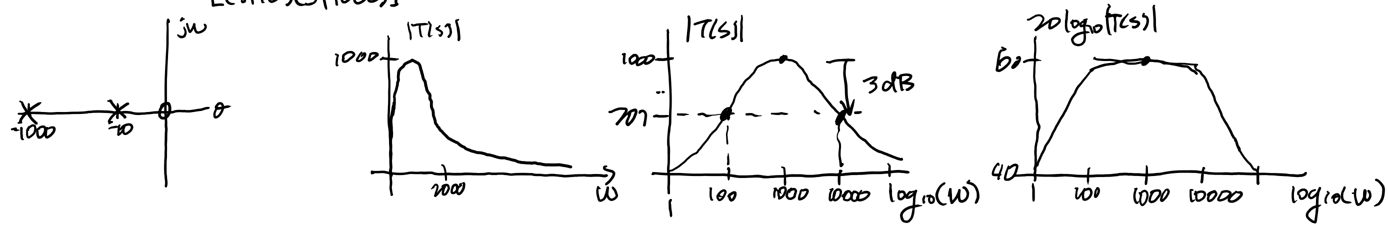
ex. $T(s) = \frac{s}{(s+10+j100)(s+10-j100)}$



ex. $T(s) = \frac{s}{(s+5+j100)(s+5-j100)}$



ex. $T(s) = (1000)\left[\frac{1000s}{(s+10)(s+1000)}\right]$



For any transfer function $T(s)$, if we want to bode plot for positive $j\omega$ axis (evaluate $s = 0 + j\omega$), we can write:

$$T(s) = T(j\omega) = \underbrace{M(\omega)}_{\text{magnitude of the transfer function}} e^{\underbrace{j\phi(\omega)}_{\text{phase of the transfer function}}}$$

$$\ln|T(j\omega)| = \ln|M(\omega)| + j\phi(\omega)$$

$$20\log_{10}|T(j\omega)| = 20\log|M(\omega)|$$

Since transfer function takes the form of polynomials / polynomials,
we can simplify as:

$$T(s) = K\frac{(s+\omega_{z1})(s+\omega_{z2})\cdots}{(s+\omega_{p1})(s+\omega_{p2})\cdots}$$

$$= K\frac{(j\omega+\omega_{z1})(j\omega+\omega_{z2})\cdots}{(j\omega+\omega_{p1})(j\omega+\omega_{p2})\cdots}$$

$$= K\frac{M_{z1}(\omega)e^{j\arctan(\frac{\omega}{\omega_{z1}})}\,M_{z2}(\omega)e^{j\arctan(\frac{\omega}{\omega_{z2}})}\cdots}{M_{p1}(\omega)e^{j\arctan(\frac{\omega}{\omega_{p1}})}\,M_{p2}(\omega)e^{j\arctan(\frac{\omega}{\omega_{p2}})}\cdots}$$

$$= K\frac{M_{z1}(\omega)M_{z2}(\omega)\cdots}{M_{p1}(\omega)M_{p2}(\omega)\cdots}\,e^{j(\arctan\frac{\omega}{\omega_{z1}}+\cdots-\arctan(\frac{\omega}{\omega_{p1}})\cdots)}$$

where $M_{z1}(\omega) = \sqrt{\omega^2+\omega_{z1}^2}\cdots$

AMPLITUDE BODE PLOT:

$$20\log_{10}|T(s)| = 20\log_{10}(K) + 20\log_{10}|M_{z1}(\omega)| + \cdots - 20\log_{10}|M_{p1}(\omega)|\cdots$$

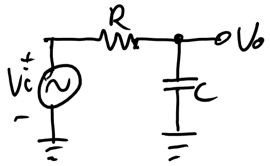
PHASE BODE PLOT:

$$\phi(\omega) = \underbrace{\tan^{-1}\left(\frac{\omega}{\omega_{z1}}\right)+\cdots-\tan^{-1}\left(\frac{\omega}{\omega_{p1}}\right)\cdots}_{\text{should add up to } 0 \text{ if } K>0,\ \pm180 \text{ if } K<0}$$

Should add up to 0 if K > 0
±180 if K < 0

Low Pass, Band Pass, & High Pass

September 25, 2017 10:45 AM



$$\frac{V_o}{V_i} = \frac{\left(\frac{1}{sC}\right)}{\left(\frac{1}{sC}+R\right)} \cdot \frac{\left(\frac{s}{R}\right)}{\left(\frac{s}{R}\right)}$$

$$= \frac{\left(\frac{1}{RC}\right)}{\left(\frac{1}{RC}+s\right)}$$

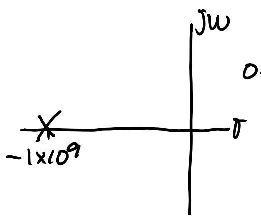
$$= \frac{\omega_p}{s+\omega_p}$$

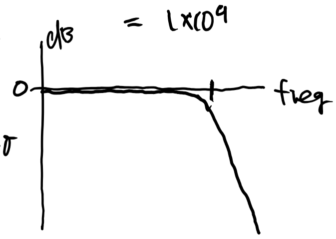
→ $\omega_p = \frac{1}{RC}$

→ circuit has a pole at $s = -\omega_p$

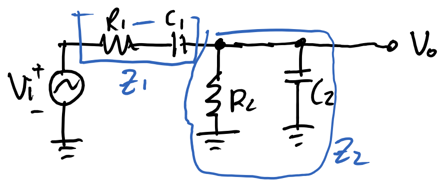
ex. if $R = 1K$, $C = 1pF$, then $\omega_p = \frac{1}{1\times10^{3} \cdot 1\times10^{-12}}$

$= 1\times10^{9}$





Given the following circuit:



Band pass filter when $C_1 \gg C_2$

Lo-pass filter when $C_2 \gg C_1$

Transfer function: $\frac{V_o}{V_i} = \frac{Z_2}{Z_1+Z_2}$, $Z_1 = R_1 + \frac{1}{sC_1}$, $Z_2 = R_2 \,//\, \frac{1}{sC_2}$

$$\rightarrow \frac{V_o}{V_i} = \frac{\left(\frac{1}{R_1C_2}\right)s}{s^2+\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}+\frac{1}{R_1C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$$

This circuit has one zero (at zero) and two poles.

Using the $\underbrace{\text{quadratic formula}}$ to solve the denominator into: $(s+\omega_{p1})(s+\omega_{p2})$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\rightarrow s_{1,2} = \frac{-\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}+\frac{1}{R_1C_1}\right)\pm\sqrt{\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}+\frac{1}{R_1C_1}\right)^2-4(1)\left(\frac{1}{R_1R_2C_1C_2}\right)}}{2(1)}$$

We can make a few approximations:

① Since $C_1 \gg C_2$ for band pass, then $\frac{1}{R_1C_2}+\frac{1}{R_1C_1} \approx \frac{1}{R_1C_2}$

$$\approx \frac{-\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}\right)\pm\sqrt{\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}\right)^2-\frac{4}{R_1R_2C_1C_2}}}{2}$$

$$\rightarrow \left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}\right)^2-\frac{4}{R_1R_2C_1C_2}$$

$$\hookrightarrow \left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}\right)^2 - \frac{4}{R_1R_2C_1C_2}$$

$$= \left(\frac{1}{R_2C_2}\right)^2 + \frac{2}{R_1R_2C_2^2} + \left(\frac{1}{R_1C_2}\right)^2 - \frac{4}{R_1R_2C_1C_2}$$

② since $C_1 \gg C_2$, then $\frac{2}{R_1R_2C_2^2} \gg \frac{4}{R_1R_2C_1C_2}$

$$\approx \frac{-\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}\right) \pm \sqrt{\left(\frac{1}{R_2C_2}\right)^2+\frac{2}{R_1R_2C_2^2}+\left(\frac{1}{R_1C_2}\right)^2}}{2}$$

$$\approx -\frac{1}{2}\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}\right) \pm \frac{1}{2}\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}\right)$$

$S_1 \approx 0$, $\quad S_2 = -\left(\frac{1}{R_2C_2}+\frac{1}{R_1C_2}\right)$

↳ low freq pole is not exactly 0, so we'll have to come back to this. (ω_{p1})

↳ $$\omega_{p2} = \frac{1}{R_2C_2}+\frac{1}{R_1C_2}$$

$$= \frac{1}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_1}\right)$$

$$= \frac{1}{C_2}\cdot\frac{1}{R_1//R_2}$$

$$\boxed{\omega_{p2} = \frac{1}{C_2(R_1//R_2)}}$$

Knowing ω_{p2}, we can find ω_{p1}:

denominator: $(s+\omega_{p1})(s+\omega_{p2}) = s^2 + (\omega_{p1}+\omega_{p2})s + \omega_{p1}\omega_{p2} = 0.$

equating w/ denominator of the transfer function

$$s^2 + (\omega_{p1}+\omega_{p2})s + \omega_{p1}\omega_{p2} = s^2 + \left(\frac{1}{R_1C_1}+\frac{1}{R_1C_2}+\frac{1}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}$$

$$\omega_{p1}\omega_{p2} = \frac{1}{R_1R_2C_1C_2}$$

$$\omega_{p1} = \frac{1}{\omega_{p2}R_1R_2C_1C_2}$$

$$\approx C_2(R_1//R_2)\left(\frac{1}{R_1R_2C_1C_2}\right)$$

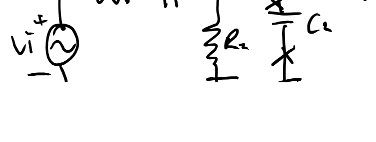
$$\approx \frac{R_1//R_2}{R_1R_2C_1}$$

$$\omega_{p1} \approx \boxed{\frac{1}{C_1(R_1+R_2)}}$$

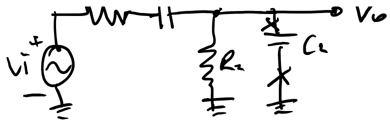
In conclusion: $\omega_{p1} \approx \frac{1}{C_1(R_1+R_2)}$

$\omega_{p2} \approx \frac{1}{C_1(R_1//R_2)}$

Notice the ω_{p1} is the $\frac{1}{\tau_{oc}^{C_1}}$, where $\tau_{oc}^{C_1}$ is found by replacing C_2 with O.C.

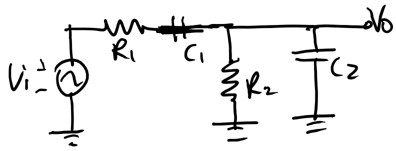


$\tau_{oc}^{C_1} = (R_1+R_2)C_1$



$\tau_{oc}^{C_1} = (R_1 + R_2)C_1$

Similarily, ω_{p2} is $\frac{1}{\tau_{sc}^{C_2}}$, where $\tau_{sc}^{C_2}$ is found by replacing C_1 with S.C.



$\tau_{sc}^{C_2} = (R_1 // R_2)C_2$

Frequency Response

September 25, 2017 5:40 PM

Recall Transfer Function from last part (with approx.)

$$\frac{V_o}{V_i} = \frac{\left(\frac{s}{R_1C_2}\right)}{\left(s+\frac{1}{(R_1+R_2)C_1}\right)\left(s+\frac{1}{(R_1/\!/R_2)C_2}\right)}$$

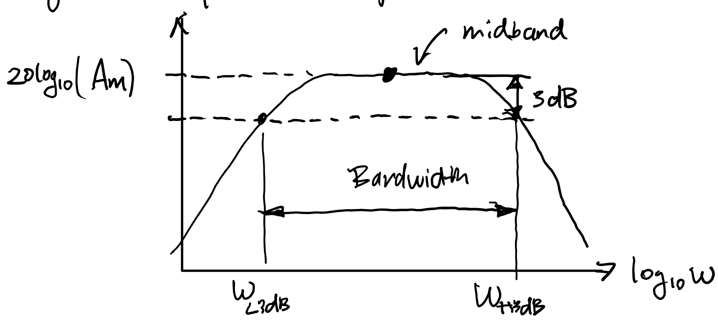
We can split into three parts: Low Freq Response, Midband, High Freq Response

$$\rightarrow = \underbrace{\left(\frac{R_2}{R_1+R_2}\right)}_{A_M}\underbrace{\left(\frac{s}{s+\omega_{p1}}\right)}_{F_L(s)}\underbrace{\left(\frac{\omega_{p2}}{s+\omega_{p2}}\right)}_{F_H(s)}$$

$F_L(s)$ takes the form $\frac{(s+\omega_{z1L})(s+\omega_{z2L})\dots}{(s+\omega_{p1L})(s+\omega_{p2L})\dots}$

$F_H(s)$ takes the form $\frac{\omega_{p1H}\omega_{p2H}\cdots(s+\omega_{z1H})(s+\omega_{z2H})\dots}{\omega_{z1H}\omega_{z2H}\cdots(s+\omega_{p1H})(s+\omega_{p2H})\dots} = \frac{\left(1+\frac{s}{\omega_{z1H}}\right)\left(1+\frac{s}{\omega_{z2H}}\right)\dots}{\left(1+\frac{s}{\omega_{p1H}}\right)\left(1+\frac{s}{\omega_{p2H}}\right)\dots}$

Magnitude Response (of a generic bandpass filter)



Finding Cutoff Frequency

If locations of poles & zeros are known:

$$\omega_{L3dB} = \sqrt{\omega_{p1L}^2 + \omega_{p2L}^2 \cdots - 2\omega_{z1L}^2 - 2\omega_{z2L}^2 \cdots}$$

$$\omega_{H3dB} = \sqrt{\left(\frac{1}{\omega_{p1H}}\right)^2 + \left(\frac{1}{\omega_{p2H}}\right)^2 \cdots - 2\left(\frac{1}{\omega_{z1H}}\right)^2 - 2\left(\frac{1}{\omega_{z2H}}\right)^2 \cdots}$$

$$= \sqrt{\tau_{p1H}^2 + \tau_{p2H}^2 \cdots - 2\tau_{z1H}^2 - 2\tau_{z2H}^2 \cdots}$$

If locations of poles & zeros are not known

then use method of SC and OC time constants.

Method of SC & OC Time Constants

$$F_L(s) = \frac{(s+\omega_{z1L})(s+\omega_{z2L})\cdots}{(s+\omega_{p1L})(s+\omega_{p2L})\cdots}$$

$$= \frac{s^{n_L} + a_1 s^{n_L-1} + \cdots a_{n_L}}{s^{N_L} + b_1 s^{N_L-1} + \cdots b_{N_L}}$$

* If one of the pole freq is > any other pole / zero freq. Then we can make the following approximation:

$$\approx \frac{s^{n_L}}{s^{N_L} + b_1 s^{N_L-1}}$$

$$F_L(s) \approx \frac{s}{s+b_1}$$

where $b_1 = \omega_{p1L} + \omega_{p2L} + \cdots$

$$= \frac{1}{\tau_{sc1}} + \frac{1}{\tau_{sc2}} + \cdots$$

$$\omega_{L3dB} \approx b_1 = \sum_{i=1}^{N} \frac{1}{C_i R_{is}}$$

$$F_H(s) = \frac{\left(1+\frac{s}{\omega_{z1H}}\right)\left(1+\frac{s}{\omega_{z2H}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1H}}\right)\left(1+\frac{s}{\omega_{p2H}}\right)\cdots}$$

$$= \frac{1 + c_1 s + \cdots c_{m_H} s^{m_H}}{1 + d_1 s + \cdots d_{M_H} s^{M_H}}$$

* if one of the pole freq is < any other pole / zero freq, Then we can approximate:

$$F_H(s) \approx \frac{1}{1+d_1 s}$$

where $d_1 = \frac{1}{\omega_{p1H}} + \frac{1}{\omega_{p2H}} + \cdots$

$$= \tau_{oc1} + \tau_{oc2} + \cdots$$

$$\omega_{H3dB} \approx \frac{1}{d_1} = \frac{1}{\sum_{i=1}^{M} C_i R_{io}}$$